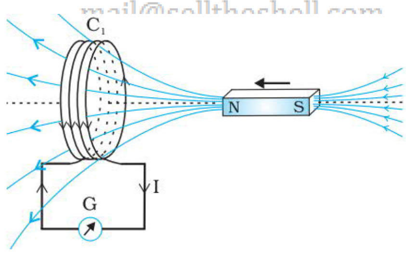
# CHAPTER 6: ELECTROMAGNETIC INDUCTION

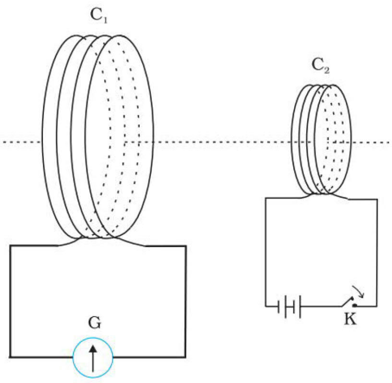


- ***Experiments of Faraday & Henry***

Observations:




(a) Galvanometer needle deflects indicating the flow of current, when the magnet is moved towards the coil.
(b) When the magnet is moved away, the needle deflects in the opposite direction, indicating that current flow in the opposite direction.
(c) When the magnet is held stationary and solenoid (coil) is moved, similar observations are noted.
(d) When there is no relative motion between the magnet and the coil, no deflection is observed.
(e) Larger the relative motion, more is the deflection.
(f) When the magnet is replaced by current carrying solenoid, same observations are noted.
(g) When soft iron (ferromagnet) is introduced as a core, greater is the deflection.
(h) When the key is closed, the galvanometer shows momentary deflection. When the current is constant, there is no deflection. If the key is open, the galvanometer shows momentary deflection, in the opposite direction.




(i) In general, it is observed that whenever the magnetic flux linked with the coil changes with time, an emf is induced in the coil, resulting in an induced current.

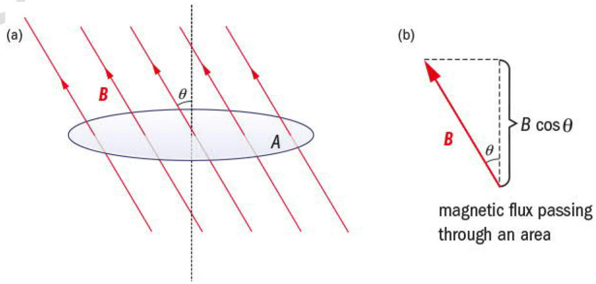
- ***Magnetic Flux ( $\Phi_B$ )***

Magnetic flux is the measure of the total magnetic field lines passing through a given area. In other words, the number of magnetic field lines crossing any area normally is defined as magnetic flux ($\Phi_B$) through the area. It is a scalar quantity. The SI unit for magnetic flux is weber (Wb) or Tm$^2$.

Mathematically, magnetic flux is defined as,

$$\Phi_B = \oint \vec{B}\,.\vec{ds} \quad or \quad \Delta\Phi_B = \vec{B}.\vec{\Delta S}$$

$$or \quad \Delta\Phi_B = B\,\Delta S\,\cos\theta$$




- ***Faraday's Laws of Electromagnetic Induction***

***First law*** - Whenever magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux lasts.

***Second law*** - The magnitude of emf induced in a closed circuit is equal to rate of change of magnetic flux linked with the circuit.

$$\varepsilon = \frac{d\Phi}{dt}$$

For N turns, it becomes, $\varepsilon = N\,\dfrac{d\Phi}{dt}$

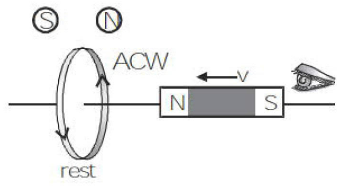
- ***Lenz's Law*** - Lenz's law states that the polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.

Mathematically,

$$\varepsilon = -N\,\frac{d\Phi}{dt} \text{ -----(1)}$$

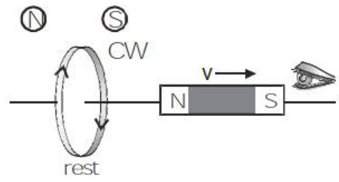
The negative sign indicates the concept of Lenz law.

For example, when the north pole of a bar magnet is moved towards a coil at rest, the side of the coil facing the magnet becomes north and a current flow in the anti-clockwise direction, when viewed from the magnet. This is because, as the magnet is moving towards the coil, the magnetic field and hence the magnetic flux through the coil increases. According to Lenz's law, the current induced in the coil should oppose this increase in flux. For this to happen, the current flows ACW through the coil and the side facing the magnet becomes north. (In simple terms N of the coil opposes (repels) the motion of the magnet)



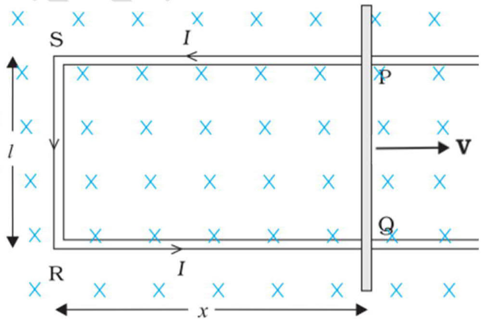

(Coil face behaves as North pole to opposes the motion of magnet.)

When the south pole of a bar magnet is moved towards a coil, the side of the coil facing the magnet becomes south and a current flow in the clockwise (CW) direction, when viewed from the magnet. This is because, as the magnet is moving away from the coil, the magnetic field and hence the magnetic flux through the coil decreases. According to Lenz's law, the current induced in the coil should oppose this decrease in flux. For this to happen, the current flows CW through the coil and the side facing the magnet becomes south. (In simple terms S of the coil opposes (attracts) motion of the magnet)




(Coil face behaves as South pole to opposes the motion of magnet.)

- ***Motional EMF*** - Consider a straight conductor PQ moving with a velocity $v$ in the rectangular loop PQRS in a uniform magnetic field $B$, perpendicular to the plane of the system.
Thus, the rectangle PQRS forms a closed circuit enclosing a varying area $A$ due to the motion of the rod PQ.
The magnetic flux $\Phi_B$ enclosed by the loop PQRS is given by,




$$\Phi_B = \vec{B}.\vec{A} = B\,A\cos\theta \quad (\theta = 180)$$
$$\Phi_B = -B\,A = -B\,l\,x \quad (A = l\,x)$$

From Faraday's law, we have,

$$\varepsilon = -\frac{d\Phi_B}{dt} = B\,l\,\frac{dx}{dt} \quad (B\ \&\ l\ are\ constants)$$
$$\varepsilon = B\,l\,v \text{ ------ (2)}$$

***Explanation of motional emf using Lorentz magnetic force***

*When the conductor PQ is moved, the electrons inside the conductor experiences a force and drifts towards the end Q. In this way, the sides P and Q becomes positive and negative respectively. Through the external loop, current flows as shown in the figure. Note that the current through the conductor is from Q to P.*

*The magnetic force on the conductor is given by,*

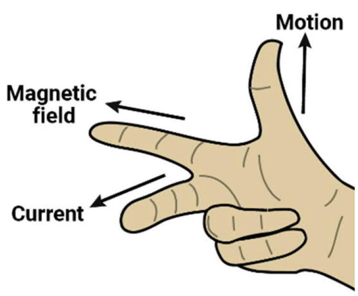
$$\vec{F} = I\,(\vec{l}\ X\ \vec{B}) \text{ -----(3)}$$

*The direction of this force is towards left, opposite to the applied force which is towards right. Its magnitude is given by,*

$$F = I\,l\,B\,sin\theta \quad (angle\ \theta\ between\ l\ and\ B\ is\ 90^o)$$
$$F = I\,l\,B \text{ -----(4)}$$

***Fleming's Right-Hand Rule***

*The direction of current through a conductor moving in a magnetic field can be found by the Fleming's right-hand rule. Hold the right-hand forefinger, middle finger and the thumb at right angles to each other. If the forefinger represents the direction of the magnetic field, the thumb points in the direction of motion or applied force, then the middle finger points in the direction of the induced current.*



***Energy Conservation – Lenz law***



Lenz law is based on the law of conservation of energy. Consider the resistance of the arm PQ be $R$ and the other sides having negligible resistance. Consequently, the current through the loop is,

$$I = \frac{\varepsilon}{R} = \frac{B\,l\,v}{R}$$

Power loss as Joule's heat in the loop is,

$$P_H = I^2 R = \frac{B^2 l^2 v^2}{R}$$

The force $F$ required to move the conductor (arm PQ) with a velocity $v$ is same as the magnetic force in magnitude, which is given by,

$$F = I\,l\,B$$

$$F = \left(\frac{B\,l\,v}{R}\right) lv = \frac{B^2 l^2 v}{R}$$

The mechanical power needed to move the conductor is given by,

$$P_M = \vec{F}.\vec{v} = F\,v\,\cos\theta$$



$$P_M = F\,v = \frac{B^2 l^2 v^2}{R}$$

$$P_H = P_M$$

Which shows that the mechanical energy used to move the arm is transformed as heat, in accordance with the law of conservation of energy.

Also, let $\Delta q$ is the charged accumulated in the loop PQRS when the change in flux is $\Delta\Phi_B$. By Faraday's law,

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = I\,R$$

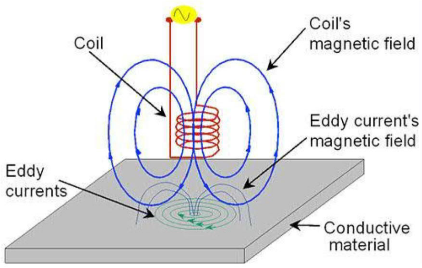
$$\frac{\Delta\Phi}{\Delta t} = \frac{\Delta q}{\Delta t} R$$

$$\Delta q = \frac{\Delta\Phi}{R} \quad \text{-----(5)}$$



- ***Eddy Currents (Foucault Currents)***

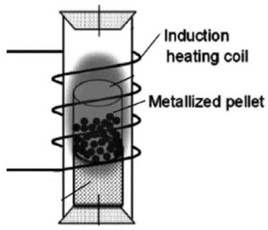
When the magnetic flux linked with a bulk conductor (e.g. metal block) varies with time, induced current loops are produced within the block. These currents lead to the loss of energy in the form of heat. The direction of current and its magnetic field will be in such that it opposes the change in the magnetic flux, in accordance with Lenz's law.




***Applications of Eddy Currents***

*Eddy currents are used to advantage in certain applications such as:*

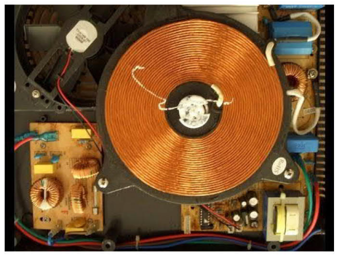
(a) ***Induction Furnace*** – *Induction furnace can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.*




(b) ***Magnetic braking in trains*** – *Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth.*

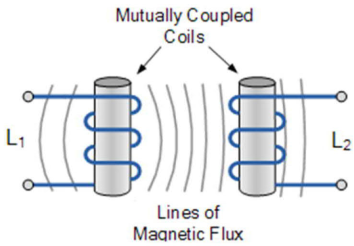
(c) ***Diathermy –*** *Eddy currents are used for deep heat treatment of the human body. Eddy currents can be used to heat the localized tissues in the human body.*

(d) ***Induction Cook-tops –*** *These cook-tops use an alternating current to induce eddy currents in the cooking utensil. The eddy currents produce heating in the metal utensil. The vessel or pan heats up without the top of the stove getting hot.*



➢ **Inductance**

Consider two inductors (coils) $L_1$ and $L_2$, placed at some distance apart. Let a current $I$ flow through $L_1$ and the flux linked with $L_2$ be $\Phi_B$. Clearly the flux $\Phi_B$ is directly proportional to the current $I$.




If there are $N$ turns in $L_2$,

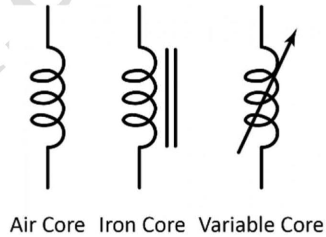
$$N\,\Phi_B \propto I$$

$$N\,\Phi_B = K\,I$$

Where $K$ is a constant known as Inductance. It's S.I. unit is henry (H). Inductance depends on :



(a) no. of turns in the coil
(b) area of cross-section
(c) length of the coil
(d) separation between the coils
(e) material of the core




Inductor Symbols

***Self Induction and Self Inductance***

Self induction is the phenomenon of emf being induced in a coil due to varying flux by its own current.

Measure of this ability to induce emf in a coil due to varying flux is called self inductance. Consider an inductor, consisting of $N$ turns, carrying a time varying current. The time varying flux through the coil is directly proportional to the current $I$.

$$N\,\Phi_B \propto I$$

$$N\,\Phi_B = L\,I$$

Where $L$ is a constant known as self inductance (or coefficient of self induction) of the coil. Differentiating () w.r.t. to time, we get,

$$N\frac{d\Phi_B}{dt} = L\,\frac{dI}{dt}$$

$$-\,\mathcal{E} = L\,\frac{dI}{dt}$$

$$\mathcal{E} = -\,L\,\frac{dI}{dt} \quad \text{--------(6)}$$

Let $\frac{dI}{dt} = 1, \quad |\mathcal{E}| = L$

*Self-inductance is numerically equal to the emf induced when the rate of change of current is unity.*

Consider an air core solenoid (inductor) of length $l$, area of cross-section $A$, carrying a current $I$. Let '$N$' be the number of turns and '$n$' the number of turns per length.

$$N = n\,l$$

The total flux through the coil is given by,

$$N\,\Phi_B = L\,I \quad \text{---- (7)}\,;$$ where $L$ is the self inductance of the coil.

$$N\,\Phi_B = (n\,l)(\mu_o\,n\,I)(A)$$

$$N\,\Phi_B = \mu_o\, n^2\, A\, l\, I \text{ ----(8)}$$



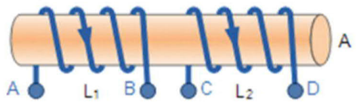
Comparing (7) and (8) $\qquad L = \mu_o\, n^2\, A\, l$ ---- (9)

If there is a core of relative permeability $\mu_r$ introduced to the inductor, the self-inductance given by (9) becomes,

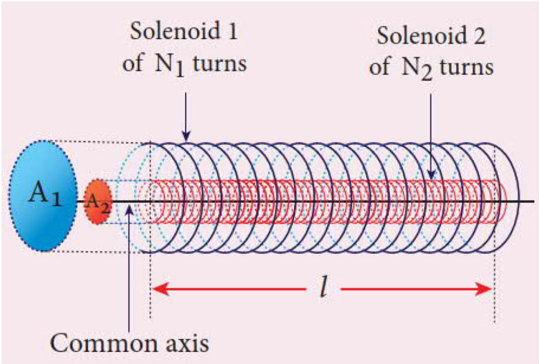
$$L = \mu_o\, \mu_r\, n^2\, A\, l \text{ ---- (10)}$$

➢ ***Mutual Induction and Mutual Inductance***

It is the phenomenon in which an emf is induced in a coil due to a varying flux produced by a neighboring coil.




Consider two coaxial solenoids $S_1$ and $S_2$, having radii $r_1$ and $r_2$ ($r_1 > r_2$), area of cross section $A_1$ and $A_2$, having the same length $l$.




| Geometry | $S_1$ | $S_2$ |
|---|---|---|
| Radius | $r_1$ | $r_2$ |
| No. of turns | $N_1$ | $N_2$ |
| No. of turns/length | $n_1$ | $n_2$ |
| Area of Cross-section | $A_1$ | $A_2$ |
| Length | $l$ | $l$ |



Let a current $I_1$ flows through $S_1$. The total flux through $S_2$ is proportional to the current $I_1$.

i.e. $\quad N_2\,\Phi_2 \;\alpha\; I_1$

$N_2\,\Phi_2 = M_{12}\, I_1$ ------(11) ; where $M_{12}$ is a constant known as the mutual inductance of $S_1$ w.r.t $S_2$.

(11) could be also expressed as,

$$N_2\,\Phi_2 = (n_2\, l)(\mu_o\, n_1\, I_1)(A_2) \quad \left\{\begin{matrix} \text{Note that flux through } S_2 \text{ is due to the} \\ \text{magnetic field is produced by } S_1 \end{matrix}\right\}$$

$$N_2\,\Phi_2 = \mu_o\, n_1\, n_2\, \pi\, r_2^{\,2}\, l\, I_1 \text{ ---- (12)}$$

Comparing (11) and (12), we get,

$$M_{12} = \mu_o\, n_1\, n_2\, \pi\, r_2^{\,2}\, l \text{ ---- (13)}$$

Similarly, if there is a current $I_2$ through $S_2$, the total flux through $S_1$ is,

$$N_1\,\Phi_1 \;\alpha\; I_2$$

$$N_1\,\Phi_1 = M_{21}\, I_2 \text{ ---- (14)}$$

Proceeding as above, we get,

$$N_1\,\Phi_1 = (n_1\, l)(\mu_o\, n_2\, I_2)(A_1)$$

$$N_1\,\Phi_1 = \mu_o\, n_1\, n_2\, \pi\, r_2^{\,2}\, l\, I_2 \text{ ---(15);}$$

*Note that the magnetic field produced by $S_2$ is confined within $S_2$. There is no magnetic field in the space between $S_1$ and $S_2$. Therefore, the effective area of $S_1$ having flux is $\pi\, r_2^{\,2}$ and not $\pi\, r_1^{\,2}$.*

Comparing (14) and (15), we get,

$$M_{21} = \mu_o\, n_1\, n_2\, \pi\, r_2^{\,2}\, l$$

$$M_{21} = M_{12} \quad (= M)$$

In general,

$$N_1\,\Phi_1 = M\, I_2 \quad \text{and} \quad N_2\,\Phi_2 = M\, I_1$$

Differentiating the above two equations and using Faraday's law, we get, $\varepsilon_1 = -M \frac{dI_2}{dt}$ and $\varepsilon_2 = -M \frac{dI_1}{dt}$



Which clearly shows that the emf induced in a coil is due to the varying current in the neighboring coil.

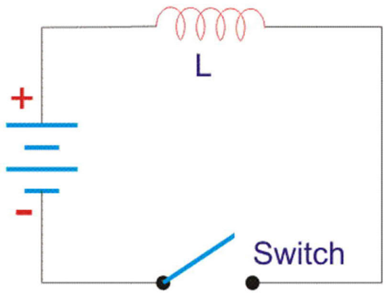
If there are currents flowing through both the coils, the induced emf in a coil will be due to both self induction and mutual induction. In such a case, the two equations mentioned above becomes,

$$\varepsilon_1 = -\ L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} \quad \text{and}$$

$$\varepsilon_2 = -\ L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

- ***Energy Stored by an Inductor***

Consider an inductor of inductance $L$, connected across a battery of potential difference $V$, through a switch. When the switch is closed, a current starts flowing through the circuit. As per the Lenz law, the induced emf across the inductor opposes the increase in current and it takes some time for the current to reach its maximum value $I$. Let $'i'$ be the instantaneous current through the inductor. The magnitude of induced emf across the inductor is given by,






$$|\varepsilon| = L \frac{di}{dt}$$

The power is given by,

$$P = i\ |\varepsilon|$$

$$\frac{dW}{dt} = i\, L \frac{di}{dt}$$

$$dW = i\, L\, di$$

the total work done is given by,

$$W = L \int_0^I i\, di$$

$$W = L \left[\frac{i^2}{2}\right] = \frac{1}{2} L\, I^2 \text{ ---(16)}$$

Applying the work - energy principle, the work done by the cell is same as the energy gained by the inductor.

$$U = \frac{1}{2} L\, I^2 \text{ ----(17)}$$

The energy stored by the inductor per unit volume is defined as the energy density($u$).

$$u = \frac{U}{volume}$$

From (), the inductance L is given by,

$$L = \mu_o n^2 A\, l$$

Also, the magnetic field by the solenoid is

$$B = \mu_o\, n\, I \quad \text{or} \quad I = \frac{B}{\mu_o\, n}$$

Substituting the relations for $L$ and $I$ in (17), we get,

$$U = \frac{1}{2} (\mu_o n^2 A\, l) \frac{B^2}{\mu_o^2 n^2}$$

$$\frac{U}{A\, l} = \frac{1}{2} \frac{B^2}{\mu_o}$$

The term $'A\,l\,'$ is the volume of the inductor. Therefore, the term of the LHS is the energy density $u$.



$$u = \frac{1}{2}\frac{B^2}{\mu_o} \text{ ------(18)}$$

It shows that an inductor stores energy using a magnetic field, similar to a capacitor stores energy using a magnetic field.

- ***AC Generator***

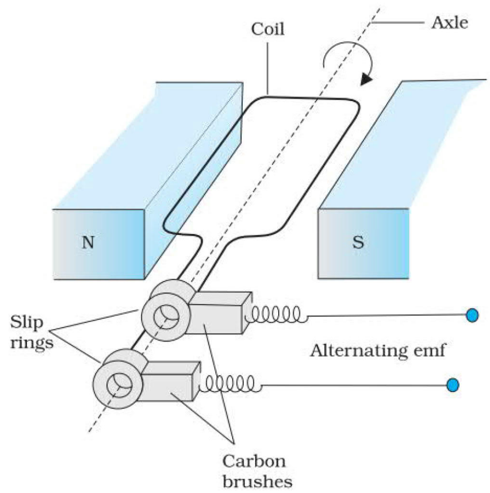
It is a device that converts mechanical energy to electrical energy. It consists of a coil, which is rotated in a magnetic field, using some external source. Since the flux linked with the coil changes, an emf is induced in the coil.

Consider a rectangular coil of N turns, area A, kept in a uniform magnetic field B as shown in the figure. The coil rotates with an angular velocity ω about its axis.

The flux linked with the coil is given by, $N\,\Phi_B = N B A \cos\theta = N\,B\,A\cos\omega t$

According to Faraday's law, the emf induced at that instant is given by,






$$\varepsilon = -N\frac{d}{dt}(\Phi_B)$$
$$\varepsilon = -N\frac{d}{dt}(BA\cos\omega t)$$
$$\varepsilon = -N\,B\,A\frac{d}{dt}(\cos\omega t)$$
$$\varepsilon = -N\,B\,A\,\omega\,(-\sin\omega t)$$

The max. value of $\sin\omega t = 1$, Therefore, $'N\,B\,A\,\omega\,'$ represents the maximum (peak) emf $\varepsilon_o$.

$$\varepsilon = \varepsilon_o \sin\omega t \text{ ---- (19)}$$

It is seen that the induced emf varies as sine function of the time angle $\omega t$. The graph between induced emf and time angle for one rotation of the coil will be a sine curve and the emf varying in this manner is called sinusoidal emf or alternating emf.