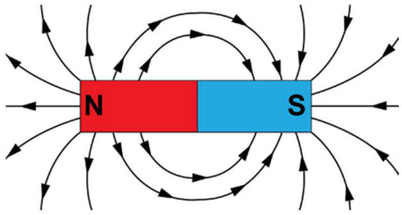
CHAPTER 5: MAGNETISM & MATTER

- ***The Bar Magnet***



(a) When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet.

(b) Like poles (N-N or S-S) repel, unlike poles (N-S) attract.

(c) We cannot isolate the north, or south pole of a magnet. Unlike electric charges, isolated magnetic north and south poles known as magnetic monopoles do not exist.

(d) The earth behaves as a bar magnet with the magnetic field pointing approximately from the geographic south to the north.

(e) The magnetic field by a bar magnet is similar to that of a current carrying solenoid.

- ***Magnetic Field lines***

Magnetic field lines are a visual tool used to represent magnetic fields. They describe the direction of the magnetic force on a north monopole at any given position. In other words, the magnetic field lines are a visual and intuitive realisation of the magnetic field. Their properties are:

(a) The magnetic field lines of a magnet (or a solenoid) form continuous closed loop, unlike electric field lines.

(b) Magnetic field lines always emerge from the north pole and terminate at the south pole, outside a magnet.

(c) The tangent at a point on the field line represents the direction of the net magnetic field at that point.

(d) The magnetic field lines never intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

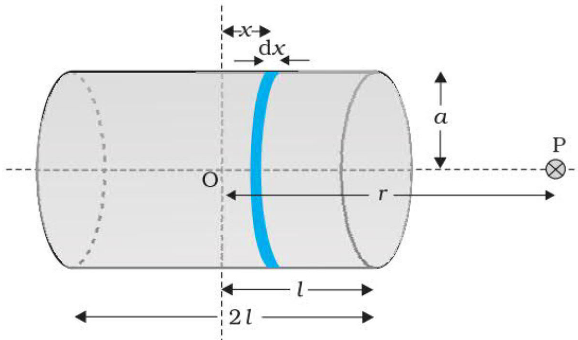
(e) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field. It means that the closeness of magnetic field lines represents the relative strength.

- ***Bar Magnet as an equivalent solenoid***

Consider a solenoid of length $2l$ carrying a current I, radius a, consisting of n turns per unit length. P is a point, at a distance r from the centre O of the solenoid. Consider a circular element of thickness dx, consisting of ndx turns, at a distance x from its centre.

The magnetic field at the point P, due to the circular element is given by,

$$dB = \frac{\mu_0 n\,dx\,I a^2}{2[(r-x)^2 + a^2]^{3/2}}$$



The magnetic field at the point by the solenoid is therefore given by,

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{l} \frac{dx}{[(r-x)^2 + a^2]^{3/2}}$$

For a point P far from the solenoid ($r >> a,\ r >> x$), the above equation reduces to,

$$B = \frac{\mu_0 n I a^2}{2 r^3} \int_{-l}^{l} dx \;=\; \frac{\mu_0 n I}{2} \frac{2 l a^2}{r^3}$$

$$B = \frac{\mu_0}{4\pi} \frac{2m}{r^3}$$ ---- (1); (Using the relations $\pi a^2 = A,\ n2l = N,\ NIA = m$)

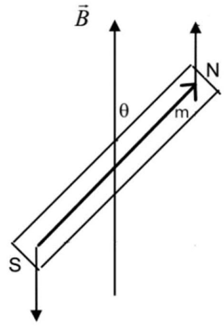
(Which is analogous to the electric field at a point on the axis of an electric dipole). Proceeding along the same line, the magnetic field at a point P, at a distance x, on the equatorial plane of a solenoid is

$$B = \frac{\mu_o}{4\pi}\frac{m}{r^3} \text{ ---- (2)}$$

Since a solenoid and a bar magnet are equivalent, equations (1) and (2) could be used to find the magnetic field by a bar magnet.

- ***Potential Energy of a magnetic dipole suspended in a uniform magnetic field***

A bar magnet or a current carrying loop is a magnetic dipole. Consider a magnetic dipole placed in a uniform magnetic field such that its magnetic dipole moment makes an angle $'\theta'$ with the magnetic field. The torque on the dipole is given by,



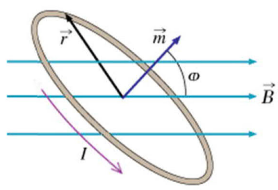
$$\vec{\tau} = \vec{m} \; X \; \vec{B}$$
$$\tau = m\,B\sin\theta$$

The work done by the external torque to rotate the dipole by an angle $d\theta$ is given by,

$$dW = \tau\, d\theta$$
$$dW = m\,B\sin\theta\, d\theta \text{ ---- (3)}$$

$\therefore$ the net work done $W = \int_{\theta_i}^{\theta_f} m\,B\, \sin\theta\, d\theta$

$$W = mB\int_{\theta_i}^{\theta_f} \sin\theta\, d\theta$$
$$W = -mB\,[\cos\theta_f - \cos\theta_i]$$



The work done is same as the energy gained by the system,

$$\Delta U = W$$
$$\Delta U = -mB\,[\cos\theta_f - \cos\theta_i] \text{ --- (4)}$$

We consider* the potential energy of the dipole to be zero at an angle $\theta = 90^o$. Let $\theta_i = 90^o$ and $\theta_f = \theta$.

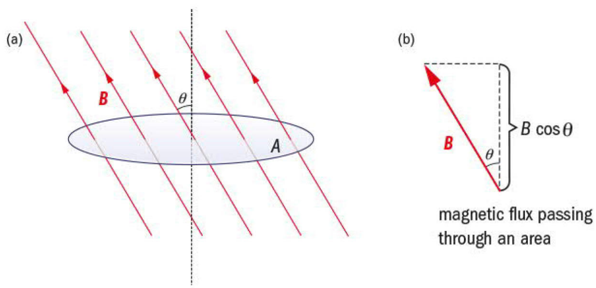
$$\Delta U = U = -m\,B\,[\cos\theta - \cos 90]$$
$$U = -m\,B\,\cos\theta \text{ ---- (5)}$$

To recapitulate, for a dipole in a uniform electric field,

$Angle\ (\theta)$	$Net\ Force\ (F)$	$Net\ Torque\ (\tau)$	$Potential\ Energy$	$Known\ as$
0	0	0	$-mB$	*Stable Equlibrium*
90^o	0	mB	0	$-$
180^o	0	0	mB	*Unstable Equlibrium*

- ***Magnetic Flux (Φ_B)***

Magnetic flux is the measure of the total magnetic field lines passing through a given area. In other words, the number of magnetic field lines crossing any area normally is defined as magnetic flux (Φ_B) through the area. It is a scalar quantity. The SI unit for magnetic flux is weber (Wb) or Tm². Mathematically, magnetic flux is defined as,



magnetic flux passing through an area

$$\Phi_B = \oint \vec{B}.\vec{ds} \quad or \quad \Delta\Phi_B = \vec{B}.\vec{\Delta S}$$
$$or \;\; \Delta\Phi_B = B\,\Delta S\,\cos\theta$$

- ***Gauss Law***

The net magnetic flux through any closed surface is zero.

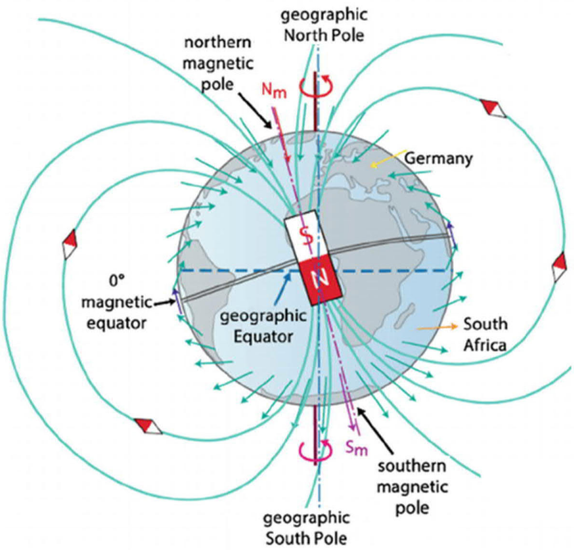
Mathematically,

$$\Phi_B = 0 \qquad or \qquad \oint \vec{B}.\vec{ds} = 0$$

Since magnetic field lines form a closed loop, the number of magnetic field lines entering and leaving a closed surface is always the same so that the net flux is zero.

- ***Geomagnetism - Earth's magnetic field and magnetic elements***

Earth is a bit like a bar magnet, with north and south poles that represent opposing magnetic polarities and invisible magnetic field lines encircling the planet between them.



It is believed that the Earth's magnetic field is generated in the fluid outer core by a self-exciting dynamo process. Electrical currents flowing in the slowly moving molten iron generate the magnetic field.

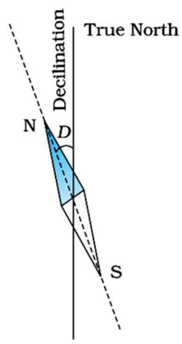
Magnetic poles of the earth are the points on the surface of the Earth where a freely suspended bar magnet or a magnetic needle orients itself vertically.

Geographical Meridian: A vertical plane passing through the point joining the geographical north and south poles.

Magnetic Meridian: A vertical plane passing through the point joining the magnetic north and south poles.

There are three quantities required to specify the magnetic field of the Earth on its surface, which are often called as the elements of the Earth's magnetic field. They are :

(a) Angle of declination or Magnetic declination (D) – The angle between the geographical meridian and magnetic meridian at a place.



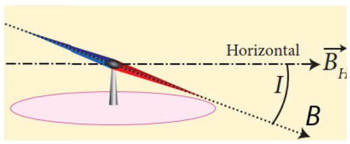
(b) Angle of dip or magnetic dip or inclination (I) – The angle that the axis of freely suspended magnetic needle placed along the magnetic meridian makes with the horizontal.

(Angle of dip is the angle that the Earth's magnetic field at a place makes with the horizontal)

Let $\boldsymbol{B}$ be the net Earth's magnetic field at any point on the surface of the Earth. $\boldsymbol{B}$ can be resolved into two perpendicular components.

Horizontal component $B_H = B \cos I$

Vertical component $B_V = B \sin I$



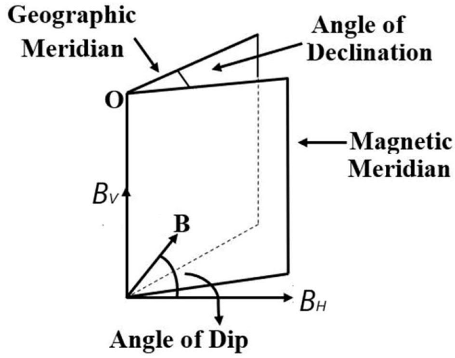
Squaring and adding, we get,

$$B = \sqrt{B_H^2 + B_V^2}$$

Dividing, we get,

$$\tan I = \frac{B_V}{B_H}$$

At the magnetic equator the Earth's magnetic field is parallel to the surface of the Earth (i.e., horizontal) which implies that the needle of magnetic compass rests horizontally at an angle of dip, $I = 0$



At magnetic poles

The Earth's magnetic field is perpendicular to the surface of the Earth (i.e., vertical) which implies that the needle of magnetic compass rests vertically at an angle of dip, $I = 90^o$

Angle of dip is measured using a dip circle which consists of a magnetic needle which is allowed to freely rotate on a vertical plane.

(c) The horizontal component of the Earth's magnetic field (B_H)