Moving Charges & Magnetism

Magnetic Effects of Current

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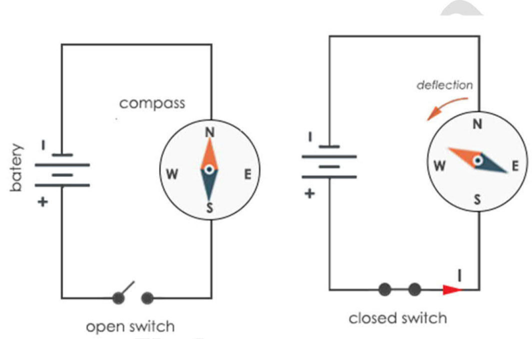
CHAPTER 4 : MOVING CHARGES & MAGNETISM

Introduction :

In the previous chapter we have seen that a difference in potential could create an electric current, which flows from higher potential to a lower potential region. In the year 1820, Hans Oersted discovered that moving charges or currents produced a magnetic field in the surrounding space. In this chapter, we will see how magnetic field exerts forces on moving charged particles, like electrons, protons, and current-carrying wires. We shall also learn how currents produce magnetic fields.

Oersted Experiment (1820)

- He observed that a compass placed near to a current carrying wire shows deflection
- The deflection of the compass needle increased with increase in the current through the wire and decreased with increase in the distance between the compass and the wire.
- When the direction of the current is reversed, the needle deflected in the opposite direction.



- For a straight current carrying wire, the magnetic field lines are circles in the transverse plane, concentric with the wire.

Lorentz Force

An electric charge will experience an electric force in a region of electric field. Similarly, a moving charge will experience a magnetic force in a region of magnetic field. So, for a charge 'q' moving with a velocity 'v' in the presence of both electric and magnetic field would experience a net force given by,

$$\vec{F} = \vec{F}_{ele} + \vec{F}_{mag}$$
$$\vec{F} = q\vec{E} + q\left(\vec{v} \times \vec{B}\right)$$
$$\vec{F} = q[\vec{E} + \left(\vec{v} \times \vec{B}\right)] \text{ ----- (1)}$$

The force given by (1) is called the Lorentz force, where the magnetic force is given by,

$$\vec{F}_{mag} = q\left(\vec{v} \times \vec{B}\right) = q\,v\,B\sin\theta \text{ (magnitude)}$$

Case 1 : If 'v' $=$ 0, i.e. the charge is at rest.

$\vec{F}_{mag} = 0$ (only a moving charge experience a magnetic force)

Case 2 : If $\theta = 0$ *or* 180°

$\vec{F}_{mag} = 0$ (if the charge moves along the field, the mag. force is zero)

Case 3 : If $\vec{F}_{mag}$ is perpendicular to $\vec{v}$, the power delivered as well as the work done by the magnetic force is zero, if the magnetic field is uniform.

$$power = \frac{Work\ done}{time} = \frac{\vec{F}_{mag} \cdot \vec{d}}{t}$$
$$= \frac{F_{mag}\ d\cos\Phi}{t} = 0$$

For e.g., when a charge undergoes uniform circular motion in a plane perpendicular to a uniform magnetic field, there is no work done on the charge and thus no change in the kinetic *energy. The magnetic field just changes the direction of velocity, keeping its magnitude unchanged.*

Magnitude of magnetic field at a place is defined as the force experienced by a unit charge moving with unit velocity perpendicular to the magnetic field. The SI unit of magnetic field is ***tesla (T).***

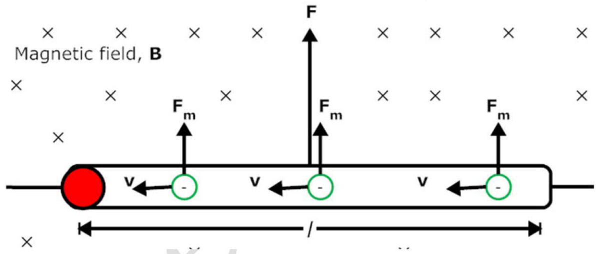
1 tesla is the field at a place if a charge of 1 coulomb moving with a velocity of 1 m/s, perpendicular to the field experiences a force of 1newton.

Since *tesla* is a large quantity, a smaller unit *gauss* (G) is often used. (1G = 10^{-4} T)

❖ ***Force on a current carrying straight wire in a uniform magnetic field***

Consider a straight wire of length $'l'$ and cross-section$'A'$, carrying a current $'I'$, placed in a magnetic field $'B'$. Let $'n'$ be the number of electrons per unit volume of the wire.

According to the Lorentz force, the magnetic force on an electron is given by,



$$\vec{F}_{mag} = \vec{f} = -e\left(\vec{v}_d \, X \, \vec{B}\right)$$

Also, the volume of the conductor = $A\,l$

∴ the number of electrons in the wire = $n\,A\,l$

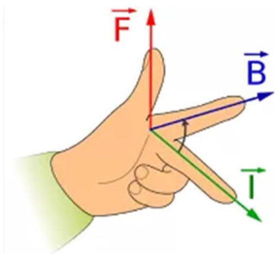
∴ the force acting on the conductor

$$\vec{F} = (\,n\,A\,l\,)\left(-e\left(\vec{v}_d \, X \, \vec{B}\right)\right)$$

which is written as $\vec{F} = n\,e\,A\,v_d\,(\,\vec{l}\;X\,\vec{B}\,)$ (Considering $'l'$ as a vector, having a magnitude equal to the length of the wire and ***direction same as that of the current.***

$$\therefore \vec{F} = I\,(\,\vec{l}\;X\,\vec{B}\,) \text{ ------ (2)}$$

The direction of force can be found by using the *right-hand rule* or by using *Fleming's left-hand rule.*

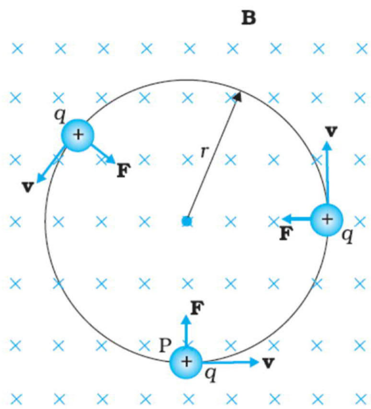


Q. A beam of α – particles projected along the x - axis experiences a force due to magnetic field along the y – axis. What is the direction of the magnetic force ? *(Ans: - z axis)*

Q. A straight horizontal conducting rod of length 0.5 m and mass 50 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero? Ignore the mass of the wires and take g = 10 ms^{-2}. *(Ans : 0.2 T)*

❖ ***Motion of a charge in a magnetic field***

Consider a charge $'q'$, of mass $'m'$, moving with a velocity $'v'$ enter a region of uniform magnetic field $'B'$, perpendicular to the field. Inside the magnetic field region the charge would experience a magnetic force perpendicular to its velocity. This force provides the centripetal force so that the charge undergoes a uniform circular motion.



$$\text{i.e. } F_{mag} = F_{cen}$$

$$q\,v\,B = \frac{m\,v^2}{r} \qquad (\,F_{mag} = q\,v\,B,\;\; as\,\theta = 90\,)$$

$\therefore \; r = \dfrac{m\,v}{q\,B}$ ----- (3), which shows that the radius of the circular path is directly proportional to its speed.

$\therefore \; r = \dfrac{p}{q\,B} = \dfrac{\sqrt{2\,m\,K}}{q\,B}$ ----(4); where K is the KE of the particle.

From (3), we get,

$$v = \frac{q\,B\,r}{m} \text{ ------ (5)}$$

Let 'T' is the time period of the charged particle, which is given by,

$$T = \frac{2\,\pi\,r}{v} \text{ ----- (6)}$$

Substituting (5) in (6), we get,

$T = \dfrac{2\,\pi\,m}{q\,B}$ ------ (7) and the angular frequency 'ω' is given by,

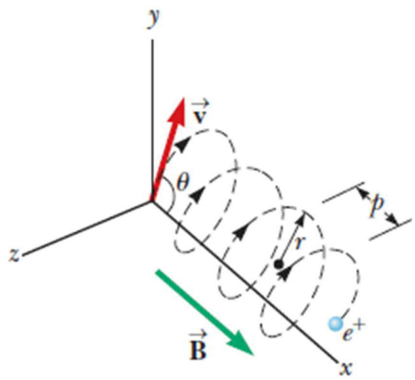
$$\omega = \frac{q\,B}{m} \text{ ------(8)} \qquad \left(\because \omega = \frac{2\,\pi}{T}\right)$$

From (6) and (7) we can see that T and ω are independent of the velocity of the charged particle as well as to the radius of the circular path. It depends only on the charge to mass $\left(\frac{q}{m}\right)$ ratio of the particle.

If the charge enters the magnetic field at an angle θ, its velocity can be resolved into two components one parallel($(v\,\cos\theta)$ and other perpendicular $(v\,\sin\theta)$ to the magnetic field. There is no force acting on the charge due to the parallel component whereas there is a force due to the perpendicular component. The resultant path of the charge is a helix. The radius of the helical path is given by,

$r = \dfrac{m\,v\sin\theta}{q\,B}$ and the time period is given by,

$T = \dfrac{2\,\pi\,m}{q\,B}$ (independent of θ)



The linear distance moved by the particle in one complete rotation is called '*pitch*' *(p)*, which is given by,

$$p = v\cos\theta \;\; T$$

$$p = v\cos\theta \;\; \frac{2\,\pi\,m}{q\,B}$$

Q. An electron and a proton enter a region of uniform transverse magnetic field with same momentum. Which particle has the larger orbital radius and time period ?

(Ans : same radius, proton)

Q. A proton and an α particle enter a region of uniform transverse magnetic field. Find the ratio of their radii and time periods if they enter

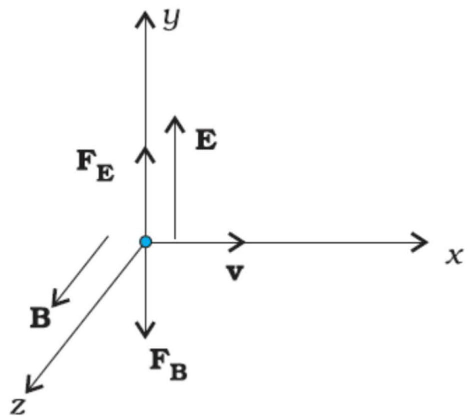
a) with same momentum b) with same velocity c) with same kinetic energy d) after being accelerated through same potential difference V

Ans :

	$\frac{r_p}{r_\alpha}$	$\frac{T_p}{T_\alpha}$
a	2 : 1	1 : 2
b	1 : 2	1 : 2
c	1 : 1	1 : 2
d	1 : $\sqrt{2}$	1 : 2

❖ **Motion of a charge in a crossed Electric and Magnetic field – Velocity Selector**

When the electric and magnetic fields are perpendicular to each other, it is called a crossed field. Consider a charge 'q' moving with a velocity 'v' along the x – axis, enter a region of crossed electric and magnetic fields as shown in the figure. Inside the region, it will experience an electric and magnetic force, so that the resultant force is given by the Lorentz equation,



$$\vec{F} = q[\vec{E} + (\vec{v} \, X \, \vec{B})]$$

$$\vec{F} = q[E\hat{j} + (v\hat{\imath} \, X \, B\hat{k})]$$

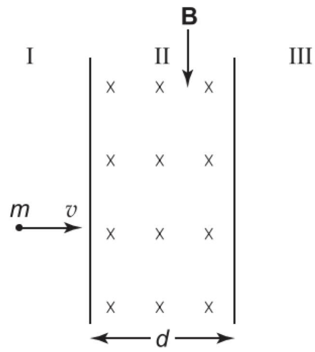
$$\vec{F} = q[E\hat{j} - (vB)\hat{j}]$$

$$\vec{F} = q[E - vB]\hat{j}$$

The net force $\vec{F} = 0$ when $E - vB = 0$ or $E = vB$ ----- (9)

For a specific value of velocity 'v', E and B are adjusted so that $E - vB = 0$. In such a case, those particles with a velocity 'v' will move undeflected whereas particles with a different velocity will get deflected.

Q. A particle of charge q and mass m moving in region I with a velocity v enters normally a region II of width d where a uniform magnetic field B (directed inwards) exists as shown in Fig. There is no magnetic field in regions I and III. What is the maximum speed (v_{max}) of the particle so that it returns back in region I ?



(Ans: $\frac{qBd}{m}$*)*

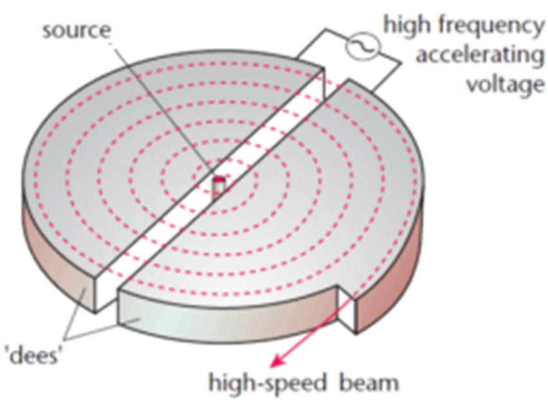
❖ ***Cyclotron***

A cyclotron is a device used to accelerate charged particles to very high kinetic energy using a crossed electric and magnetic field. It is based on the principle that,

a) the time period of a charged particle in a uniform transverse magnetic field is independent of its velocity and the radius of the orbit.
b) a charged particle is accelerated to high velocity by making it pass through a moderate electric field repeatedly with the help of a transverse magnetic field.

Working & Theory

A charged particle emitted by the source S at the centre of the *dees* accelerate towards the *dee* with opposite polarity gaining kinetic energy. Once the charge enter the dee, it moves in a semi-circular path due to the magnetic field and the absence of electric field (electrostatic shielding). The frequency of the oscillator (accelerating voltage) is adjusted in such a way that when the charge particle emerges from the dee, the polarity of the dees should be reversed. The particle again gets accelerated gaining



velocity and kinetic energy and enters the other dee. This process continues and the particles move in a spiral path inside the dees. When sufficient energy is acquired by the charged particle, they are deflected out of the cyclotron using a deflecting plate.

The centripetal force required by the charge is provided by the magnetic force. Hence,

$$q\,v\,B = \frac{m\,v^2}{r} \qquad (\,F_{mag} = q\,v\,B,\ \ as\,\theta = 90\,)$$

$$\therefore\ r = \frac{m\,v}{q\,B}\,,$$ which shows that the radius of the circular path is directly proportional to its speed.

and $$v = \frac{q\,B\,r}{m} \text{ ---- (10)}$$

Let ' T ' is the time period of the charged particle, which is given by,

$$T = \frac{2\,\pi\,r}{v}$$

$$T = \frac{2\,\pi\,m}{q\,B}$$ and the frequency $'\nu'$ is given by,

$$\nu = \frac{q\,B}{2\,\pi\,m} \text{ ----- (11)}$$

The frequency given by (10) is called the cyclotron frequency ν_C . The frequency of the oscillator ν_a is adjusted to be equal to the cyclotron frequency ν_C. i.e. $\nu_a = \nu_C$. This is called the resonance condition.

The KE of the particle at an instant is given by,

$$K = \frac{1}{2}\,m\,v^2 \text{ ----- (12)}$$

Squaring (10) and then substituting the same in (12), we get,

$$K = \frac{q^2 B^2\,r^2}{2\,m}$$

The KE of the particle emerging from the cyclotron (max. KE) is,

$$K_{Max} = \frac{q^2 B^2\,R^2}{2\,m} \text{ ------(13)}$$

which shows that the maximum energy acquired by the particles depends on the size (radius R) of the cyclotron and the strength of the magnetic field used.

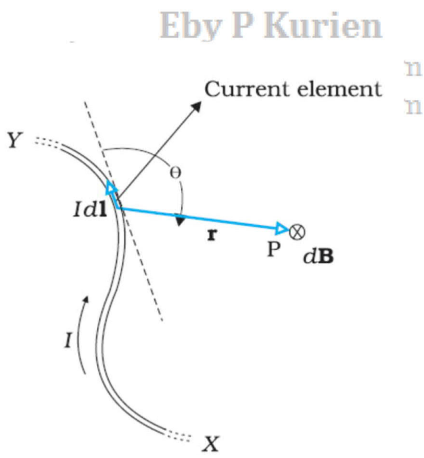
Limitations of a cyclotron

a) It cannot be used to accelerate uncharged particles like neutrons.
b) Electrons cannot be accelerated due to its small mass. The cyclotron frequency becomes very high and it becomes difficult to achieve the resonance condition.
c) Particles cannot to accelerated to very high velocities, close to the speed of light, due to the fact that the mass the particle increases significantly (relativistic effect) and breaks the resonance condition.
d) Creating a uniform magnetic field over a large region is difficult.

Q. An α–particle moving with a velocity v in a uniform magnetic field is moving in a circular path at frequency ν called the cyclotron frequency. What will be the cyclotron frequency of a proton moving with a speed 2 v in the same magnetic field ? *(Ans : 2ν)*

❖ *Biot – Savart's Law*

We have learnt that moving charges (current) creates a magnetic field. Biot – Savart's law gives the relation between current and the magnetic field it produces. Consider a finite conductor carrying a current 'I' and a point P at a distance $\vec{r}$ from a very small length dl (current element) of the conductor. Let 'θ' is the angle between $\vec{dl}$ and $\vec{r}$. The magnetic field at the point P due to the current element is given by,



$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

$$dB = \frac{\mu_o}{4\,\pi}\,\frac{I\,dl\,\sin\theta}{r^2} \text{ ----- (14)}$$

In vector form, we have,

$$\vec{dB} = \frac{\mu_o}{4\,\pi}\,\frac{I\,\vec{dl}\,X\,\vec{r}}{r^3} \text{ ----- (15)}$$

where μ_o is a constant called as the permeability of vacuum. In SI system,

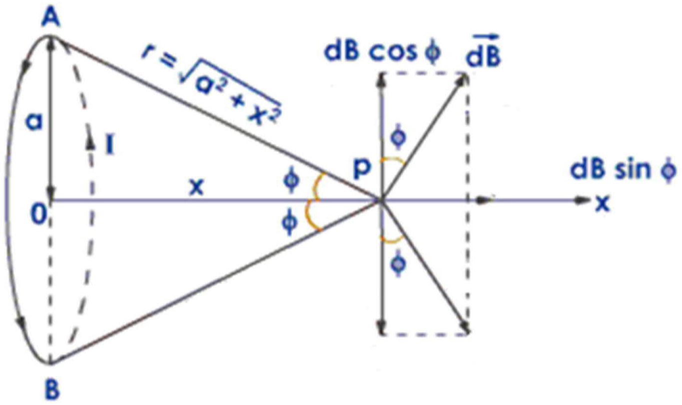
$$\mu_o = 4\pi \text{ x } 10^{-7} \text{ TmA}^{-1}$$

❖ *Magnetic Field on the axis of a circular current loop*

Consider a circular current carrying wire (loop) of radius 'a', carrying a current 'I'.

'P' is a point on the axis at a distance 'x' from the centre of the current loop. A and B are two diametrically opposite current elements 'dl'.

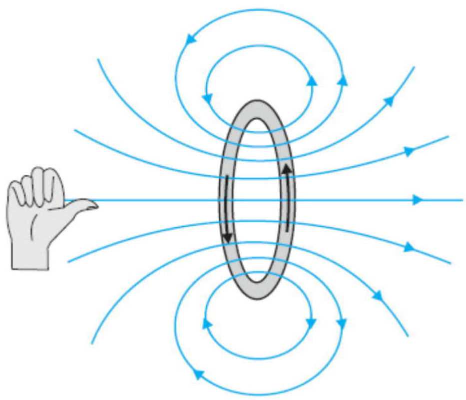
The magnitude of magnetic field at the point 'P' due to the current element at A is given by,



$$dB = \frac{\mu_o}{4\,\pi}\,\frac{I\,dl}{r^2} \qquad (\because \theta = 90\,)$$

The direction of dB is according to the right-hand rule. The magnetic field dB could be resolved into two perpendicular components, as shown in the figure.

The current element at B creates a magnetic field at 'P' with the same magnitude but with a different direction, which too could be resolved into two components. Since the cosine components cancel each other, the magnetic field at 'P' is given by,



$$B = \int_0^{2\pi a} dB \sin\Phi$$

$$B = \int_0^{2\pi a} \frac{\mu_o}{4\pi} \frac{I\, dl}{r^2} \sin\Phi$$

$$B = \frac{\mu_o}{4\pi} \frac{I \sin\Phi}{r^2} \int_0^{2\pi a} dl$$

$$B = \frac{\mu_o}{4\pi} \frac{I \sin\Phi}{r^2} 2\pi a$$

$$B = \frac{\mu_o}{2} \frac{I\, a^2}{r^3} \qquad \left(\because \sin\Phi = \frac{a}{r} \right)$$

$$B = \frac{\mu_o\, I\, a^2}{2\,(x^2 + a^2)^{3/2}} \text{ ------(16)}$$

In vector form, we have,

$$\vec{B} = \frac{\mu_o\, I\, a^2}{2\,(x^2 + a^2)^{3/2}}\, \hat{\imath}$$

If there are 'N' turns, it becomes,

$$\vec{B} = \frac{N\, \mu_o\, I\, a^2}{2\,(x^2 + a^2)^{3/2}}\, \hat{\imath}$$

Special Cases :

1. At the centre of the loop, i.e., when $x = 0$, (16) becomes,

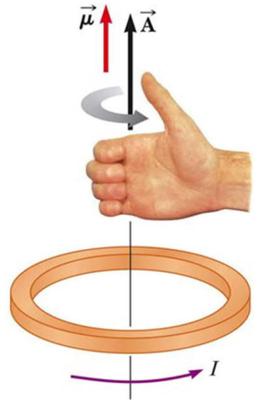
$$B_o = \frac{\mu_o\, N\, I}{2\, a} \text{ --------(17)}$$

2. Field at a far point. i.e. $x >> a$, so that a^2 can be neglected, (16) becomes,

$$B = \frac{\mu_o\, I\, a^2}{2\, x^3}$$

$$or \quad B = \frac{\mu_o}{4\pi} \frac{2\, N\, I\, \pi a^2}{x^3} \text{ ------(18)}$$

Comparing this relation with that of electric field due to a dipole we can see that the term $N\, I\, \pi a^2 = N\, I\, A$ is equivalent to the term electric dipole moment 'p' and hence it is named as the magnetic dipole moment.



❖ *Magnetic Dipole Moment ($\vec{m}$ or $\vec{\mu}$)*

The magnetic moment for a current carrying coil is given by,

$\vec{m} = N\, I\, \vec{A}$ ------- (19), where $N -$ number of turns

$I -$ current through the coil

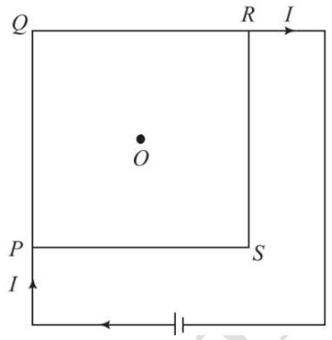
$\vec{A} -$ area vector

It's a vector quantity having a direction same as the area vector, given by the right-hand thumb rule. SI unit - Am^2 or J/T

Q. A uniform straight wire is turned into a circular wire loop of radius r. The diametrically opposite points of the loop are connected to a battery. If I is the current flowing through the battery, what is the magnetic field at centre of the loop ?

(Ans : Zero)

Q. A uniform straight wire of length L is turned into a square. The points P and R are connected to a battery as shown in Fig. If current I flow through the battery, find the magnetic field at centre O of the square. *(Ans : Zero)*



Q. Two identical coils carry equal currents and have a common centre, but their planes are at right angles to each other. What is the magnitude of the resultant magnetic field at the centre if the field due to one coil alone is B? *(Ans : $\sqrt{2}$ B)*

❖ Ampere's Circuital Law (ACL)

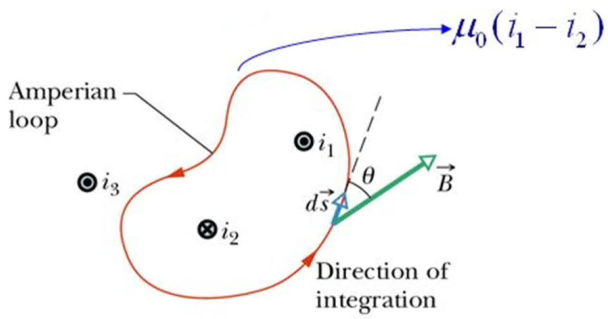
Ampere's circuital law states that the line integral of magnetic field over a closed loop is μ_o times the total current enclosed by the loop.

Mathematically,

$$\oint \overrightarrow{B}.\overrightarrow{dl} = \mu_o\, i \quad \text{--------(20)}$$

The sign convention for the current $'i'$ is given by the right-hand rule. Let the fingers of the right-hand be curled in the sense in which we traverse the loop (direction of integration), then the current in the direction of thumb is taken as positive.

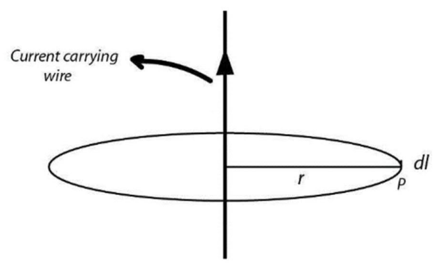
For e.g., consider a closed loop (amperian loop) enclosing two current carrying wires i_1 (outward) and i_2 (inward). The direction of integration is taken anti-clockwise. As per the right-hand thumb rule, the thumb pointing outwards represents the positive current. i.e., i_1 is positive and i_2 negative. Therefore, the RHS of (20) becomes $\mu_o\,(i_1 - i_2)$. The current i_3 has no significance since it is outside the loop.



❖ Applications of ACL

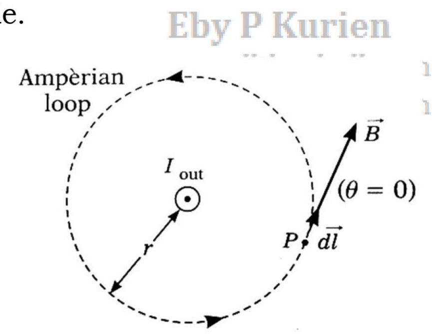
1. Magnetic field due to a straight infinite current carrying thin wire

Consider a very long, thin, straight wire carrying a current '*I*'. '*P*' is a point at a distance '*r*' from the wire. Consider a circular amperean loop of radius '*r*' in the transverse plane of the wire and concentric with the wire. By symmetry, the magnitude of magnetic field at all the points on the loop have the same value and direction is



along the tangent to the loop as given by the right-hand rule.

Applying ACL to the loop in the same sense as the field, we get,



$$\oint \vec{B}.\vec{dl} = \mu_o\, i$$

$$\oint B\, dl = \mu_o\, i \quad (\because B \text{ and } dl \text{ are parallel })$$

Since the magnetic field remains constant throughout the loop, we get,

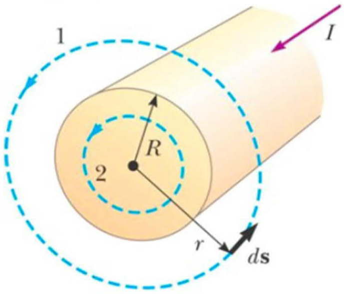
$$\therefore B \oint dl = \mu_o\, i$$

$$B\ 2\pi r = \mu_o I \quad (i = I)$$

$$\therefore B = \frac{\mu_o I}{2\pi r} \text{ ------ (21)}$$

2. *Magnetic field due to a long straight current carrying wire of radius 'R'*

Consider a current carrying long, straight thick wire of radius 'R'.



(a) *For a point outside the wire (r > R)*

Consider a point outside the wire, at a distance '*r*' from its centre. Amperean loop is circular loop of radios '*r*' (Loop 1)

From ACL,

$$\oint \vec{B}.\vec{dl} = \mu_o\, i$$

$$\oint B\, dl = \mu_o\, i \quad (\because \text{B and dl are parallel })$$

Since the magnetic field remains constant throughout the loop, we get,

$$\therefore B \oint dl = \mu_o\, i$$

$$B\ 2\pi r = \mu_o I \quad (i = I)$$

$$\therefore B = \frac{\mu_o I}{2\pi r} \text{ ------ (21)}$$

(b) *For a point inside the wire (r < R)*

Consider a point inside the wire, at a distance '*r*' from its centre. Amperean loop is circular loop of radios '*r*' (Loop 2)

From ACL,

$$\oint \vec{B}.\vec{dl} = \mu_o\, i$$

$$\oint B\, dl = \mu_o\, i \quad (\because \text{B and dl are parallel })$$

Since the magnetic field remains constant throughout the loop, we get,

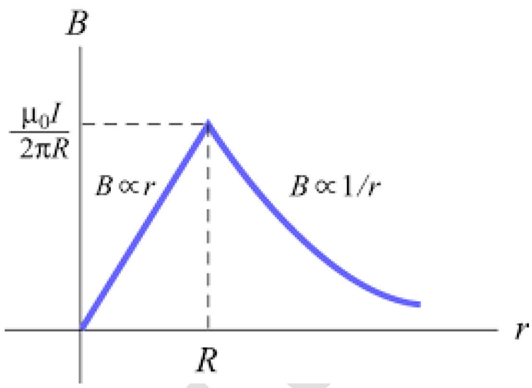
$$\therefore B \oint dl = \mu_o\, i \text{ -----(20.1)}$$

the current density, assuming to be a constant is

$$j = \frac{I}{\pi R^2}$$

∴ the current enclosed by the loop 2, is given by,

$$i = \frac{I}{\pi R^2} \pi r^2 = \frac{I r^2}{R^2}$$



∴ substituting this in (20.1), we get,

$$B \oint dl = \mu_o \frac{I r^2}{R^2}$$

$$B\, 2 \pi r = \mu_o \frac{I r^2}{R^2}$$

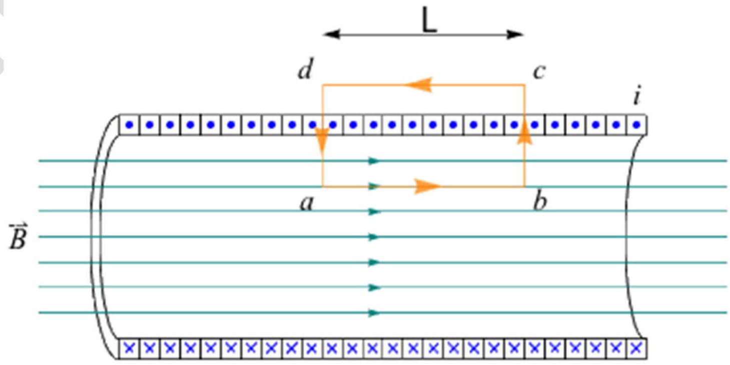
$$\therefore B = \frac{\mu_o I r}{2 \pi R^2} \text{ ------(22)}$$

At the centre of the wire ($r = 0$), $B = 0$

At the surface of the wire ($r = R$), $B = \frac{\mu_o I}{2 \pi R}$

3. *Solenoid*

It consists of a long-insulated wire wound in the form of a helix on a cylindrical core. Each turn of the solenoid can be taken to be circular for simplicity. The net magnetic field is the vector sum of fields due to all these turns. When a current is allowed to pass through a solenoid, it creates a magnetic field similar to that of a bar magnet. Consider a long solenoid, having 'n' turns per length carrying a current 'I'. The magnetic field inside the solenoid is along the axis, with a direction given by the right hand thumb rule.



Consider a rectangular amperean loop '*a b c d*' , which is partly inside as shown in the figure. Applying ACL to the loop '*a b c d*' , we get,

$$\oint \vec{B}.\vec{dl} = \mu_o i$$

$$\therefore \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl} = \mu_o i$$

The second and the fourth integral becomes zero (θ = 90). The third integral too becomes zero as the magnetic field B outside the solenoid is zero. Therefore the above equation reduces to,

$$\int_a^b \vec{B}.\vec{dl} = \mu_o\, i$$

$$\int_a^b B\ dl = \mu_o\, i \quad (\because \theta = 0)$$

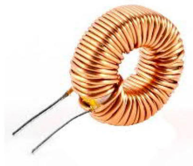
Along '$a\,b$' the magnetic field B remains a constant,

$$B\int_a^b dl = \mu_o\, i$$

$$B\,L = \mu_o\, n\, L\, I$$ (∵ the total current enclosed = no. of turns inside the amperean loop x current through each turn)

$$\therefore B = \mu_o n\, I \quad \text{----(23)}$$

4. *Toroid (Toroidal Solenoid)*

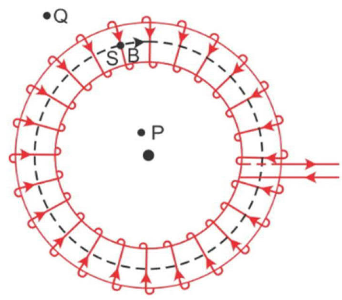


A toroid is a solenoid in the shape of a ring. For an ideal toroid, each turn is taken to be circular.

Consider a toroid having 'N' turns, 'n' turns per unit length, carrying a current 'I'. Toroid divides the space into three regions, which are,

(a) open space inside the toroid
(b) outside the toroid
(c) core of the toroid

P, Q & S are three points in these regions respectively.



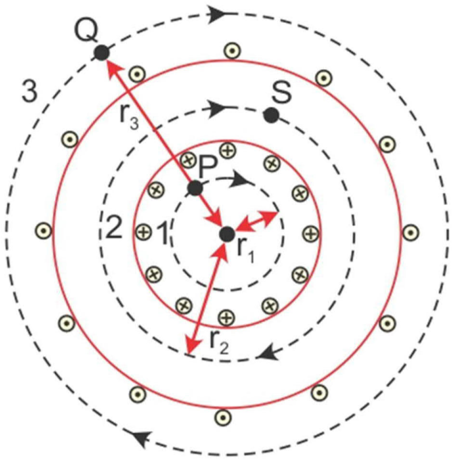
(a) Magnetic field at 'P' (open space inside the toroid)

Consider a circular amperean loop of radius r_1(loop 1), passing through the point 'P'. From ACL,

$$\oint \vec{B}.\vec{dl} = \mu_o\, i$$

$$\oint \vec{B}.\vec{dl} = 0$$, since the net current enclosed by the loop is zero.

$$\therefore B = 0$$



(b) *Magnetic field at 'Q' (outside the toroid*)

A circular amperean loop of radius r_3 (loop 3) , is passing through the point 'Q'.

From ACL,

$$\oint \vec{B}.\vec{dl} = \mu_o\, i$$

$$\oint \vec{B}.\vec{dl} = 0$$, since the net current enclosed by the loop is zero as per the sign convention.i.e.equal number of inward & outward current.

$$\therefore B = 0$$

(c) *Magnetic field at 'S' (inside the core)*

A circular amperean loop of radius r_2 (loop 2) , is passing through the point 'S'.

From ACL,

$$\oint \vec{B} . \vec{dl} = \mu_o \, i$$

$$\oint B \, dl = \mu_o \, i \qquad (\because \text{B and dl are parallel})$$

Since the magnetic field remains constant throughout the loop, we get,

$$\therefore B \oint dl = \mu_o \, i$$

$$B \; 2 \pi r_2 = \mu_o \, N \, I \quad (\text{net current } i = N \, I)$$

$$\therefore B = \frac{\mu_o N \, I}{2 \pi r_2} \text{ ----- (24)}$$

Let 'r' be the average radius of the toroid, ie, $r_2 = r$, (24) becomes,

$$\therefore B = \frac{\mu_o N \, I}{2 \pi r} \text{ ----- (25)}$$

or

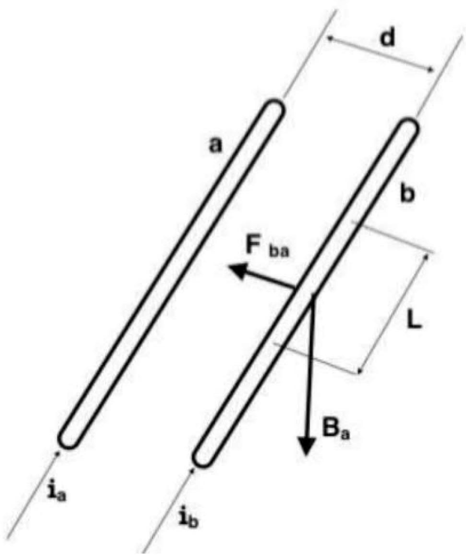
$$B = \mu_o n \, I \text{ ----- (26)}$$

In an ideal toroid, coils are circular. In reality, the turns of the toroid form a helix, there is always some magnetic field external to toroid.

Force between two parallel currents

We know that, a current current carrying conductor creates a magnetic field and a current carrying conductor in a magnetic field experiences a magnetic force. Hence two conductors placed nearby will exert a force on each other, since each of these conductor is in a region of magnetic field produced by the other.

Consider two long, parallel current carrying conductors 'a' and 'b', carrying current i_a and i_b respectively. The conductors are separated by a distance 'd'. The conductor 'a' produces same magnetic field 'B_a' at all points along the conductor 'b', in the downward direction, with a magnitude given by,



$$B_a = \frac{\mu_o \, i_a}{2 \pi d} \qquad \text{from (21)}$$

The conductor 'b' will experience a force towards the conductor 'a'. The force on a segment 'L' of conductor 'b' is,

$$F_{ba} = i_b \, L \; B_a$$

$$F_{ba} = i_b \, L \, \frac{\mu_o \, i_a}{2 \, \pi \, d}$$

$$F_{ba} = \frac{\mu_o \, i_a \, i_b \, L}{2 \, \pi \, d}$$

Similarly, the force on a segment of length 'L' on 'a' due to 'b' is given by,

$$F_{ab} = i_a \, L \, B_b$$

$$F_{ab} = \frac{\mu_o \, i_a \, i_b \, L}{2 \, \pi \, d}$$

The forces F_{ba} and F_{ab} are of equal magnitude, but of opposite direction.

i.e., $\vec{F}_{ab} = -\vec{F}_{ba}$, consistent with Newton's third law.

The force acting on the conductor per unit length, is given by,

$$f = \frac{\mu_o \, i_a \, i_b}{2 \, \pi \, d} \quad \text{---- (27)}$$

We have seen that the currents flowing in the same direction attract each other. It can be shown that currents in the opposite direction repel.

i.e. Parallel currents attract, anti-parallel currents repel

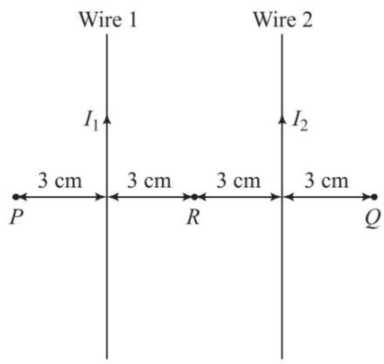
Definition of ampere

Ampere is the value of that steady current which, when maintained in each of the two very long, straight, parallel conductors of negligible cross-section, placed 1 metre apart in vacuum, would produce, on each of these conductors, a force equal to 2 x 10^{-7} N/m of length.

$$f = \frac{\mu_o \, i_a \, i_b}{2 \, \pi \, d}$$

If $i_a = i_b = 1\text{A}$, $d = 1m$, then, $f =$ 2 x 10^{-7} N/m

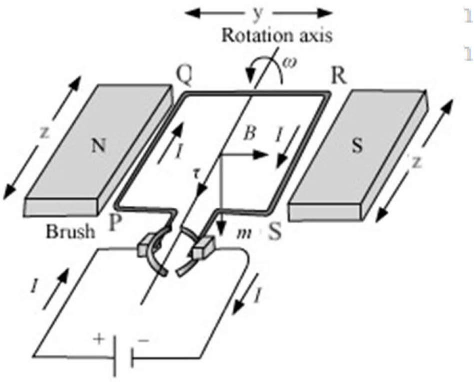
Q. Two long wires 1 and 2 carrying equal currents $I_1 = I_2 = 9A$ are held parallel to each other 6 cm apart as shown in Fig. Find the magnetic field at (a) point P, (b) point Q and (c) point R. (Ans : 8 x 10^{-5} T (outward), 8 x 10^{-5} T (inwards), Zero)



Q. Two long wires are kept 8 cm apart and carry currents of $I_1 = 2$ A and $I_2 = 10A$ in opposite directions. At what distance from wire 1 will the resultant magnetic be zero? (Ans : 2 cm)

Q. The battery of a car is connected to the motor by 50 cm long wires which are 1.0 cm apart. If the current in the wires is 200 A, find the force between the wires. Is the force attractive or repulsive. (Ans : 0.4N, Repulsive)

Torque on a rectangular loop in a uniform magnetic field



A current carrying wire in a magnetic field experiences a force. In a similar way, a current carrying loop in a magnetic field will experience a torque.

Consider a rectangular loop of length 'l' and breadth 'b' , carrying a current 'I' , placed in a magnetic field 'B'.

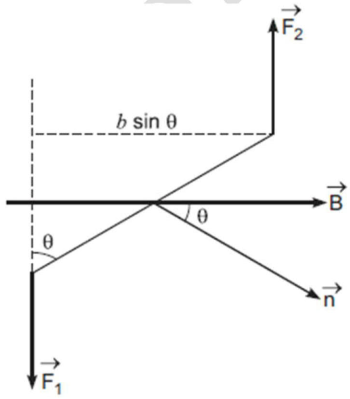
The net force acting on sides QR and PS is zero. The force acing on the side PQ is given by,

$$F_1 = I\,l\,B \qquad (l \perp B)$$

Similarly, the force acting on the side RS is given by,

$$F_2 = I\,l\,B \qquad (l \perp B)$$

These two forces having the same magnitude but opposite direction are not having the same line of action and hence constitute a couple (torque). The torque on the loop is,



$$\tau = one\ force\ X\ \perp distance\ between\ the\ forces$$

$$\tau = I\,l\,B\ (b \sin\theta)$$

$$\tau = I\,A\,B\ \sin\theta \quad (l\,b = Area\ of\ the\ loop, A)$$

$$\tau = m\,B\ \sin\theta \qquad (IA = magnetic\ moment, m)$$

In vector form,

$$\vec{\tau} = \vec{m}\ X\ \vec{B} \text{ ---- (28)}$$

Q. A bar magnet is suspended at a place where it is acted upon by two magnetic fields which are inclined to each other at an angle of 75°. One of the fields has a magnitude 2 x 10^{-2} T. The magnet attains stable equilibrium at an angle of 30° with this field. Find the magnitude of the other field. *(Ans : 10^{-2} T)*

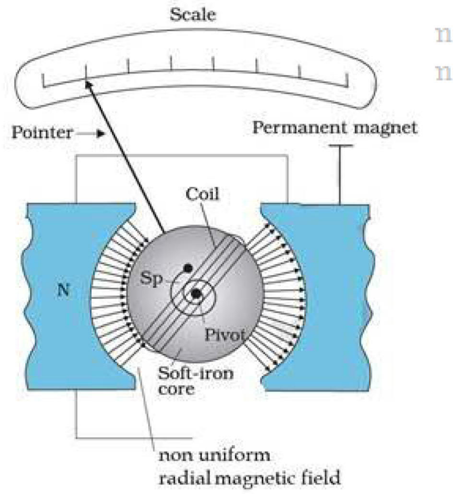
Q. A uniform wire is bent into the shape of an equilateral triangle of side a. It is suspended from a vertex at a place where a uniform magnetic field B exists parallel to its plane. Find the magnitude of the torque acting on the coil when a current I is passed through it. (Ans: $\frac{\sqrt{3}}{4}\ Ia^2B$)

Q. A particle of charge q is revolving in a circle of radius r with a constant speed v. Find the ratio of the magnitudes of magnetic moment and angular momentum of the particle.

(Ans : $\frac{q}{2m}$)

Q. A closely wound solenoid of 1000 turns and area of cross-section 5 cm^2 carries a current of 3 A. It is suspended through its centre. (a) What is the magnetic moment? (b) What is the force and torque acting on the solenoid if a uniform magnetic field of 8 x 10^{-2} T is set up at an angle of 30° with the axis of the solenoid? *(Ans: 1.5 J/T, Zero, 6 x 10^{-2} Nm)*

Moving Coil Galvanometer

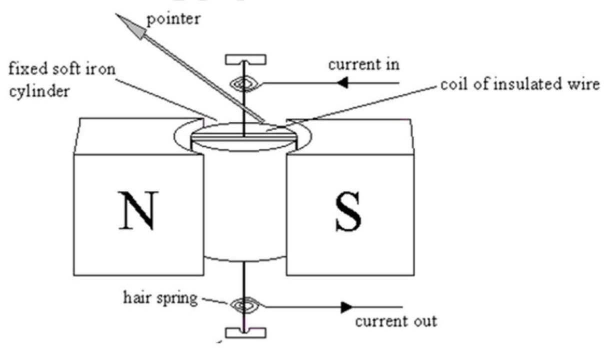


A galvanometer is a device used to detect the presence of very small current. It works on the principle that a current carrying coil (loop) placed in a magnetic field experiences a torque.

It consists of a permanent magnet with concave pole pieces and cylindrical soft iron core placed between the magnets. This arrangement produces a radial magnetic field. Due to the radial magnetic field, the torque on the coil becomes uniform, i.e. independent of the angle θ. Also, the torque on the coil is maximum as $\theta = 90^{o}$.

Copper coil is wound on an aluminium frame. (Al is used to make it a dead beat galvanometer)

Hair springs are made of phosphor bronze, which acts as current leads as well as produces a counter torque to balance the torque produced by the coil.



Theory / Working : When a current flows through the coil, it experiences a torque, given by ,

$$\vec{\tau} = \vec{m} \; X \; \vec{B}$$

$$\tau = m\,B$$

$$\tau = N\,I\,A\,B \text{ --- (29)}$$

The coil rotates causing the hairsprings to get twisted, by the way a counter torque is developed, which is proportional to the angle of twist Φ, given by,

$\tau_{spring} = k\,\Phi$ ---- (30), where k is the torsional constant of the spring.

At equilibrium,

$$\tau = \tau_{spring}$$

$$N\,I\,A\,B = k\,\Phi$$

$$I = \frac{k}{N\,A\,B}\;\Phi$$

i.e, $I \propto \Phi$; which shows that the deflection of the pointer increases with the current through the coil.

Figure of merit : It is the current per unit deflection. Unit : A/div

$$\text{i.e. } \frac{I}{\Phi} = \frac{k}{N\,A\,B} \text{ ----- (30)}$$

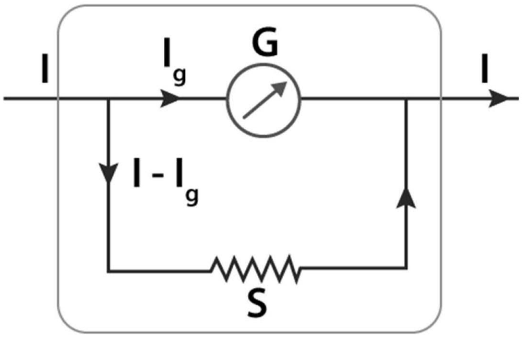
So, galvanometer is sensitive if figure of merit is small.

Ammeter is an instrument used to measure the current in a circuit. It is always connected in series. An ideal ammeter has zero resistance.

Conversion of a galvanometer to an ammeter

A galvanometer is converted to an ammeter by connecting a small value of resistance (shunt) in parallel.

Let I – Current to be measured (maximum)
I_g – Current for full scale deflection
S – Shunt resistance
G – Resistance of the galvanometer



Ammeter

Since the galvanometer and shunt are in parallel, the potential difference across the two will be the same.

$$(I - I_g)\,S = I_g\,G$$

$$\therefore S = \frac{I_g\,G}{(I - I_g)}$$

Current sensitivity of a galvanometer : It is the deflection for unit current. Unit: div/A.

$$\frac{\Phi}{I} = \frac{N\,A\,B}{k} \text{-----(31)}$$

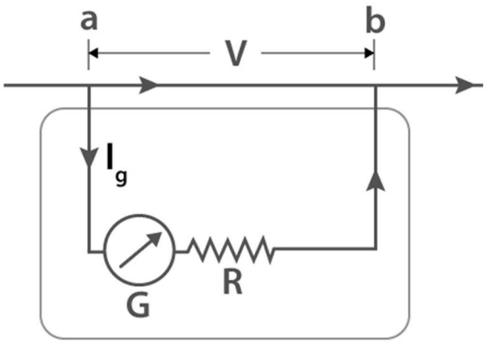
For higher current sensitivity, k should be small, N, A, B large within limits.

Voltmeter is a device which measures voltage (potential difference) across two points in a circuit. It is always connected in parallel across the two points, whose potential difference is to be measured. An ideal voltmeter has an infinite resistance.

Conversion of a galvanometer to an voltmeter

A galvanometer is converted to an voltmeter by connecting a high resistance in series.

G - resistance of galvanometer coil
I_g - full-scale current of the galvanometer
R - high resistance in series.
V - maximum voltage that needs to be measured



Voltmeter

$$V = I_g\,G + I_g\,R$$

$$\therefore R = \frac{V - I_g\,G}{I_g} = \frac{V}{I_g} - G \text{-----(32)}$$

Q. A galvanometer of resistance 20 Ω gives full scale deflection when a current of 1 mA is passed through it.
(a) How will you convert it into a ammeter that can read currents upto 1.0 A?
(b) How will you convert it into a voltmeter that can read voltages up to 5 V?
(Ans : 0.02 Ω in parallel, 4980 Ω in series)

Q. The deflection in a moving coil galvanometer falls from 50 to 10 divisions when a shunt of 12 Ω is connected across it. Find the resistance of the galvanometer. *(Ans : 48 Ω)*

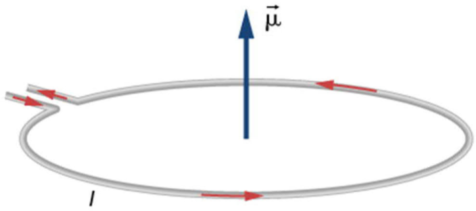
Q. A voltmeter having a resistance of 1800 Ω is used to measure the potential difference across a 200 Ω resistor which is connected to the terminals of a DC power supply having an emf of 50 V and an internal resistance of 20 Ω. What is the percentage decrease in the potential difference across the 200 Ω resistor as a result of connecting the voltmeter across it ?

(Ans : 1%)

Magnetic Dipole moment of an orbiting electron

Consider a hydrogen atom, in which an electron revolves around the nucleus (proton) in a circular orbit of radius 'r'. Let,

T – Time period of the electron
v – orbital velocity
m – mass of electron
e – charge of electron
μ_l – magnetic dipole moment



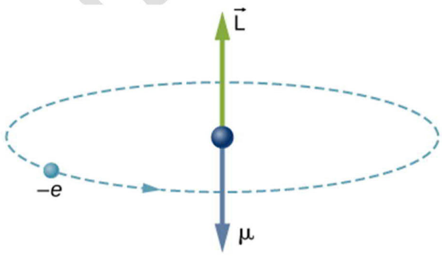
(a) Current-carrying loop

The orbiting electron is similar to a current carrying circular loop. The magnetic dipole moment is given by,

$$\mu_l = I\,A$$

$$\mu_l = \frac{e}{T}\,\pi\,r^2\ ; \qquad \because I = \frac{q}{t}$$

$$\mu_l = \frac{e\,v}{2\,\pi\,r}\,\pi\,r^2 \quad \because T = \frac{2\,\pi\,r}{v}$$



(b) Hydrogen atom

$$\mu_l = \frac{e\,v\,r}{2} = \frac{e\,m\,v\,r}{2\,m}$$

$$\mu_l = \frac{e\,L}{2\,m} \text{ ------ (33)}$$

$$\vec{\mu_l} = -\,\frac{e\,\vec{L}}{2\,m} \text{ ------ (34)}$$

From (33),

$\frac{\mu_l}{L} = \frac{e}{2\,m}$ ------ (35), is a constant called as the gyro-magnetic ratio.

From Bohr's second postulate, $L = n\,\frac{h}{2\,\pi}$, where $n = 1,2,3 \ldots$

Substituting this in (33), we get,

$$\mu_l = \frac{n\,e\,h}{4\,\pi\,m} \text{ ------ (36)}$$

For $n = 1$,

$\mu_l = \frac{e\,h}{4\,\pi\,m}$, is a constant called as the Bohr magneton. It is the smallest value of the magnetic moment of the electron. From (36) it can be seen that the magnetic dipole moment is quantised.