Current Electricity

Charges in motion

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CHAPTER 3: CURRENT ELECTRICITY

Electrostatics deals with charges at rest whereas current electricity deals with electric charges in motion. When two charged bodies having different electric potentials are joined by a conducting wire, electrons flow from the body having a lower potential to the body having a higher potential. By convention, the direction of current is taken opposite to the direction of electron flow. In other words, conventional current flows from higher potential to lower potential.

When the free charges in a conductor move in a specific direction under the action of an electric field, a current is said to be flowing in the conductor. Current which flows in the conductor for a short duration of time is called transient current. To maintain a steady current through a conductor, a potential difference has to be maintained across the conductor.

- ***Electric Current***

If 'ΔQ' amount of charge crosses an area of a conductor in time Δ't' seconds, then the average current is given by,

$I_{avg} = \frac{\Delta Q}{\Delta t}$, and the instantaneous current is given by,

$$I = \frac{dQ}{dt} \quad \text{------ (1)}$$

Current is a scalar quantity having an S.I unit ampere (A). The direction of current (conventional) is take in the direction opposite to that of the direction of electron flow.

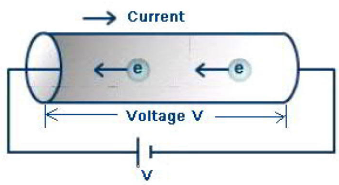
Q. The current I (in ampere) flowing in a wire varies with time t (in second) according to the equation $I = 5t + 3t^2$. Find the amount of charge which passes through a cross-section of the wire in the time interval $t = 2s$ to $t = 4s$. *(86 C)*

- ***Ohm's Law***

Ohm's law states that the current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor provided the physical conditions remain constant.

i.e. $I \alpha V$

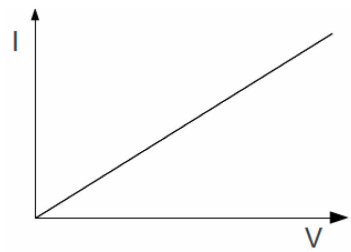
or $\frac{V}{I} = R$ ---- (2), where R is a constant known as the resistance of the conductor. It a constant for a given conductor at a constant temperature and it depends on the nature of the material and the geometrical shape of the conductor.



S.I. unit of resistance is ohm (Ω). The reciprocal of resistance is called Conductance (C)

$$\frac{1}{R} = C \quad \text{--- (3)}$$

S.I. unit of conductance : mho or Ω^{-1} or Siemens (S)



Materials which follow Ohm's law are called as ohmic materials. e.g. metallic conductors.

Those materials which don't obey Ohm's law are called non – ohmic materials. e.g. semi-conductors, electrolytes.

It is found that the resistance of a conductor is directly proportional to its length and inversely proportional to its area of cross-section.

$$\text{i.e. } R \; \alpha \; \frac{l}{A}$$

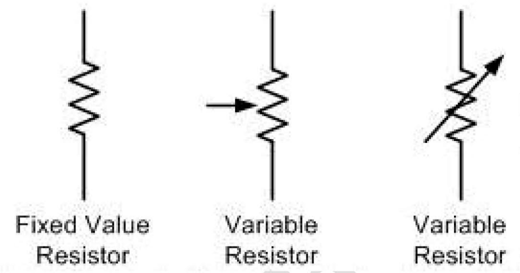
$$R = \rho \frac{l}{A}$$ ---(4); where $'\rho'$ is a constant known as the resistivity or the specific resistance of the material of the wire. It depends on the material of the wire and temperature. The S.I. unit of resistivity is ohm-meter (Ω-m)

If $A = 1\,m^2$ and $l = 1m$, from (4), we get,

$$R = \rho$$

∴ Resistivity of a material is the resistance across the length of a conductor having unit length and unit area of cross-section.

Resistor is a component which offer resistance, used in circuits to control the flow of current.



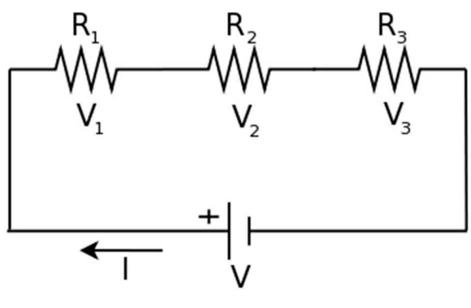
Resistor Symbol

The two most common types of resistors are

a) Carbon resistors
b) Wire wound resistors

- ***Combination of Resistors***

a) ***Resistors in Series*** – Consider three resistors R_1, R_2 and R_3 connected in series with a cell of potential difference $'V'$. Let 'R' is the equivalent resistance of the combination. In series combination of resistors, the current through the resistors will be the same whereas the applied potential difference gets divided across each resistor.



$$\therefore \; V = V_1 + V_2 + V_3$$

From Ohm's law, we get,

$$IR = I\,R_1 + I\,R_2 + I\,R_3 \quad \text{(I is a constant)}$$

$$\therefore \; R = R_1 + R_2 + R_3 \text{ ----- (5)}$$

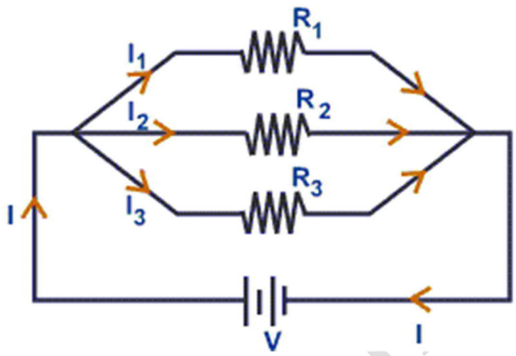
For 'n' resistors in series, the equivalent resistance is given by,

$$R = R_1 + R_2 + R_3 + \cdots + R_n \text{ ----- (6)}$$

If there are *'n'* resistors in series, each having same value *'R'*, the equivalent resistance is given by,

$$R_{equ} = nR$$

b) ***Resistors in Parallel*** – Consider three resistors R_1, R_2 and R_3 connected in parallel with a battery of potential difference $'V'$. Let 'R' is the equivalent resistance of the combination. In a parallel combination of resistors, the potential difference across each resistor will be the same whereas the total current (I) gets divided through each resistor.



$$\therefore\ I = I_1 + I_2 + I_3$$

From Ohm's law, we get,

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \qquad \text{(V is a constant)}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{ ---- (7)}$$

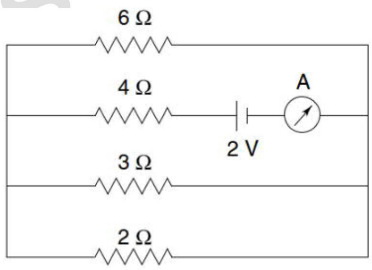
For *'n'* resistors in parallel, the equivalent resistance is given by,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \text{ ------ (8)}$$

If there are *'n'* resistors in parallel, each having value *'R'*, the equivalent resistance is given by,

$$R_{equ} = \frac{R}{n}$$

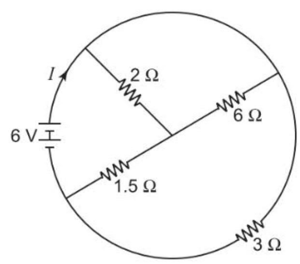
Q. *In the circuit shown in figure, what is the reading of ammeter?* *(0.4A)*



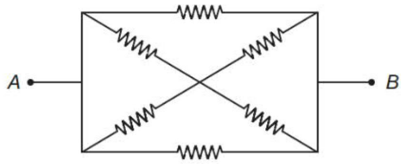
Q. *A wire has a resistance of 9Ω. It is cut into three equal pieces. Each piece is stretched uniformly to three times its original length. The three stretched pieces are then connected in parallel. Find the total resistance of the combination.* *(9Ω)*

Q. *A wire is stretched to make it 0.1% longer. What is the percentage change in the resistance?* *(0.2%)*

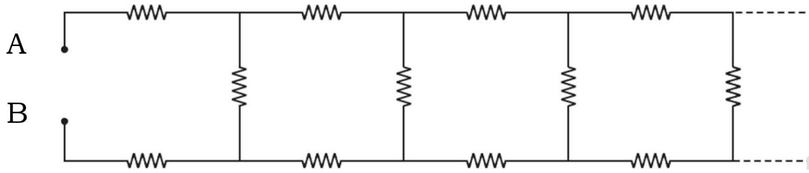
Q. *Find the value of current I in the network shown in figure.* *(Ans :4A)*



Q. Find the equivalent resistance between points A and B in the network shown in Fig. Each resistor has a resistance of 3 Ω. *(Ans: 1Ω)*

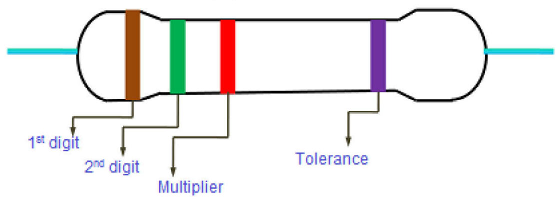


Q. Determine the equivalent resistance between the points A and B of the infinite network shown in figure. Each resistor has a resistance of 1Ω. (Ans: 2.73Ω)



➢ ***Carbon Resistors & Colour Code***

Carbon resistors are made by graphite mixed with suitable adhesive. The resistance of a carbon resistor is represented on it using a series of colour bands, where each colour represents a value. The table showing the values is as shown in the table:



Color	Digit	Multiplier	Tolerance (%)
Black	0	10^0 (1)	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
(none)			20

➢ ***Current Density*** (j)

Current density is the current passing through unit area held normal to the flow of current. It is a vector quantity having a direction of the electric field. Its S.I. unit is Am^{-2}

From Ohm's law, we have,

$$V = I\,R \quad \text{(Scalar form)}$$

Substituting (4) in the above equation we get,

$$V = I\ \rho\,\frac{l}{A} \quad \text{--- (9)}$$

The potential difference 'V' is given by,

$$V = E\,l$$

$$\therefore \text{(9) becomes, } E\,l = \rho\,\frac{I}{A}\ l$$

$$\text{or, } E = \rho\,\frac{I}{A}$$

But $\frac{I}{A}$ = current density (j).

$$E = j\,\rho \quad \text{--- (10)}$$

Or, $j = \frac{1}{\rho} E$

$$\vec{j} = \sigma \vec{E} \text{ --- (11); (Vector form of Ohm's law)}$$

where $'\sigma'$ is the conductivity of the conductor, which is the reciprocal of resistivity.

$$\sigma = \frac{1}{\rho} \text{ --- (12)}$$

- ***Drift Velocity*** (v_d)

In the absence of a potential difference, the free electrons in a conductor are in random motion so that the average velocity of an electron is zero. When a potential defence is applied across the conductor, an electric field is developed within the conductor so that the electrons experiences a force and starts moving towards one end of the conductor. During its motion it collides with other electrons and the stationary atoms and gets rebound. Due to this the electrons just drifts to one end of the conductor instead of a constant accelerated motion.

The average velocity by which the free electrons move in a conductor in the presence of an electric field is called the drift velocity.

Consider a conductor of resistance 'R', length 'l' and area of cross-section 'A'. Let 'ρ' is the resistivity of the material of the conductor and 'n' be the number of electrons per unit volume. When a potential difference 'V' is applied across the ends of the conductor, an electric field 'E' is developed within the conductor so that the free electrons experiences a force 'F', given by,

$$\vec{F} = -e\vec{E}$$

From the second law of motion, we have,

$$\vec{F} = m\vec{a}$$

From the above equations, we get,

$$\vec{a} = \frac{-e}{m}\vec{E} \quad \text{--- (13)}$$

As discussed before, in the absence of an electric field, the electrons are in random motion and hence the average velocity is zero.

$$\text{i.e. } < v_i > = 0 \quad \text{--- (14)}$$

In the presence of an electric field, the electrons drifts to one end of the conductor with an average velocity $\boldsymbol{v_d}$

From the first equation of motion,

$$v = v_i + a\,t$$

Taking the average value of the above equation, we get,

$$< v > = < v_i > + < a\,t >$$

$$< v > = 0 - < \frac{e}{m} E\,t >$$

$$< v > = 0 - \frac{e}{m} E < t > \quad \text{---- (15)}$$

The average velocity $< v >$ on the LHS is the drift velocity and the average time $< t >$ on the RHS is called as the relaxation time (τ). Relaxation time is the average time between two successive collisions.

$\therefore$ (15) becomes,

$$v_d \; = \; - \, \frac{e\,E}{m} \;\tau \quad \text{---- (16)}$$

Inside the conductor, due to repeated collisions, free charges don't have any acceleration, but only average velocity. The relaxation time $'\tau'$ depends on the nature of the material and temperature.

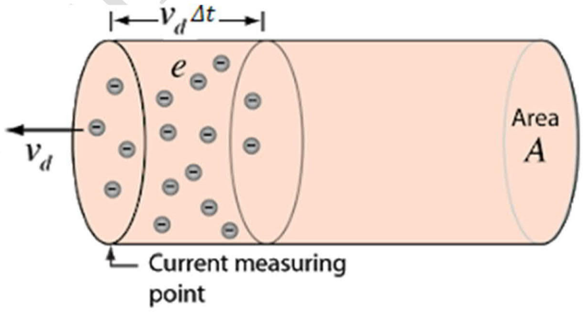
- ***Origin of resistivity***

Consider a conductor of resistance 'R', length 'l' and area of cross-section 'A'. Let 'ρ' is the resistivity of the material of the conductor and 'n' be the number of electrons per unit volume. A potential difference 'V' is applied across the ends of the conductor so that the free electrons drift with a velocity v_d.

Let Δq is the amount of charge flowing through the conductor in time Δt seconds.

$\Delta q = N\,e$, where 'N' is the number of electrons cross the cross-section of the conductor in time Δt.

$$\Delta q = (\,n\,A\;\; v_d\;\Delta t\,)\;\; e$$



$$\Delta q = n\;\;e\;\;A\;\;v_d\;\;\Delta t$$

$$\frac{\Delta q}{\Delta t} \;=\; n\;\;e\;\;A\;\;v_d$$

$$I \;=\; n\;\;e\;\;A\;\;v_d \quad \text{---- (17)}$$

or, $$\frac{I}{A} \;=\; n\;e\;\;v_d$$

$$j \;=\; n\;\;e\;\;v_d \quad \text{----- (18)}$$

Substituting the value of v_d from (16), we get,

$$j \;=\; n\;e^2\,\frac{\tau}{m}\;\;E$$

But from (11), $\vec{j} \;=\; \sigma\;\vec{E}$

Comparing the above two equations, we have,

$$\sigma \;=\; \frac{n\;e^2\;\tau}{m} \quad \text{---- (19)}$$

The resistivity 'ρ' is the reciprocal of conductivity $'\sigma'$.

$$\rho \;=\; \frac{m}{n\;e^2\;\tau} \quad \text{---- (20)}$$

The resistance and resistivity of a conductor increases with increase in temperature. This is because, as temperature increases, the free electrons acquire more thermal energy and moves faster so that the average time between two successive collisions (relaxation time) decreases. From (20) we can see that the resistivity increases with decrease in the relaxation time (τ).

Mobility (μ)

Mobility is defined as the magnitude of drift velocity per unit electric field.

$$\text{i.e. } \mu = \frac{|v_d|}{E}$$

Substituting the relation for $'v_d'$ from (16), we get,

$$\mu = \frac{e}{m}\tau \quad \text{---- (21)}$$

The S.I. unit of mobility is $m^2/V\,s$

Q. A potential difference of 0.8 V is maintained between the ends of a metal wire of length 1.0 m. The number density of free electrons in the metal is 8.0 x10^{28} per m^3 and the electrical conductivity of the metal is 6.4 x10^{7} $\Omega^{-1}m^{-1}$. Find the drift speed of electrons. *(4mm/s)*

- ***Temperature dependence of Resistivity***

As discussed earlier, the resistivity and hence the resistance of a conductor (metal) increases with increase in temperature. The variation of resistivity with temperature is approximately given by the relation,

$$\rho = \rho_o\,[1 + \alpha(T - T_o)] \quad \text{---- (22)}$$

Where,

ρ_o is the resistivity at temperature T_o
ρ is the resistivity at temperature T and
α is the temperature coefficient of resistivity of the material.

Rearranging (22) we get,

$$\rho - \rho_o = \rho_o\ \alpha(T - T_o)$$

$$\therefore \alpha = \frac{\rho - \rho_o}{\rho_o\ (T - T_o)}$$

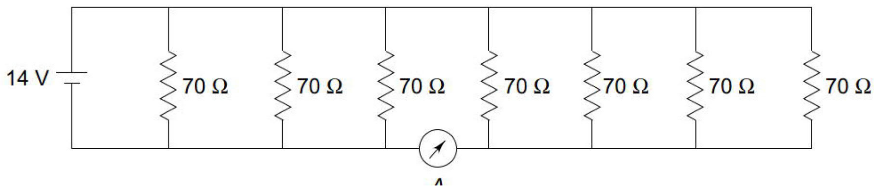
$$\text{or, } \alpha = \frac{\Delta\rho/\rho_o}{\Delta T} \quad \text{---- (23)}$$

Hence, the temperature coefficient of resistivity is defined as the fractional change in resistivity per unit change in temperature.

Q. The resistance of a heating element is 100Ω at 27°C. What is the temperature of the element if its resistance is 117 Ω. Temperature coefficient of resistance = 1.7 x 10^{-4} K^{-1}. *(1027°C)*

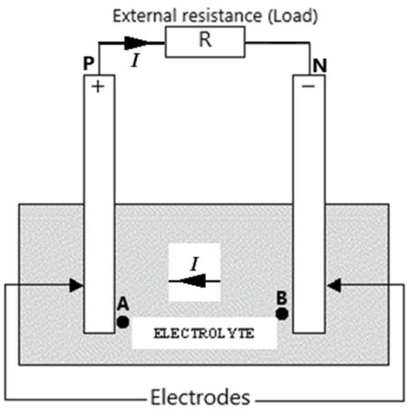
Q. The resistance of a wire is 3.00 Ω at 0°C and 3.75 Ω at 100°C. Its resistance is measured to be 3.15 Ω at room temperature. Find the room temperature. *(20°C)*

Q. Find the reading of the ammeter in the circuit shown below. *(0.8 A)*



- ***Cell, EMF and Internal Resistance***

A cell is a device which converts chemical energy to electrical energy. It maintains a potential difference between its two terminals (electrodes) A and B. As soon as the electrodes are inserted into the electrolyte, one electrode acquires a positive potential w.r.t. the electrolyte while the other acquires a negative potential w.r.t. the electrolyte. When a conducting wire is connected between the terminals of the cell, a current start to flow through the wire, from the positive to the negative terminal. It should be noted that, inside the cell, current flows from the negative to the positive terminal.



Consider two points A and B within the electrolyte, adjacent to the electrodes, as shown in the figure. The potential of the positive electrode w.r.t A is

$$V_P - V_A = V^+ \quad (> 0)$$

Similarly, the potential of the negative electrode, w.r.t B is

$$V_N - V_B = -V^- \ (< 0)$$

When there is no current through the cell, the electrolyte has the same potential throughout (i.e. $V_A = V_B$). Therefore, the potential difference between the positive and the negative electrode is given by,

$$V_P - V_N = V^+ - (-V^-)$$

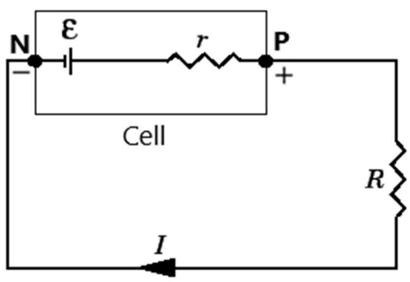
$$V_P - V_N = V^+ + V^- \text{ ------ (24)}$$

This potential difference is called as the *electromotive force (emf)* ' ε '

$$\varepsilon = V^+ + V^- \quad (> 0) \text{ ------ (25)}$$

In open circuit condition, i.e. when the current through the cell is zero, the potential difference between the terminals of the cell is called as the e.m.f of the cell. ($I = 0,\ V = \varepsilon$)

When the external circuit is closed, a current start to flow through the external circuit as well as through the cell. The current through the cell is due to the flow of electrons and ions. When they move to their respective electrodes, they collide with other ions which leads to a resistance called as the internal resistance (r) of a cell. In other words, internal resistance of a cell is the opposition to the flow of current offered by the electrolyte. It depends on the nature of the electrolyte, temperature, separation between the electrodes, etc.



From the circuit diagram, we get,

$$\varepsilon = IR + Ir$$

$$\varepsilon = V + I\,r \text{ ----- (26)}$$

$$or, \quad V = \varepsilon - I\,r \text{ ----- } (27)$$

While charging a cell, current is allowed to flow in the opposite direction. Therefore (26) becomes,

$$V = \varepsilon + I\,r \text{ ----- } (28)$$

Where V is the potential difference between the two electrodes *(same as the p.d. across R),* called as the terminal voltage.

During a short circuit, ($R = 0$), the current becomes maximum, given by,

$$I = I_{max} = \frac{\varepsilon}{r}$$

- **Electrical Energy and Power**

As discussed earlier, an electrochemical cell maintains the potential difference (V) between the two electrodes. Let Δq amount charge is transferred by the cell from one electrode to the other. The work done by the cell is given by,

$$\Delta W = V\,\Delta q$$

Dividing both sides by Δt, we get,

$$\frac{\Delta W}{\Delta t} = V\,\frac{\Delta q}{\Delta t}$$

$$P = V\,I \text{ ------- } (29)$$

Using Ohm's law, we get,

$$P = I^2 R = \frac{V^2}{R} \text{ ------- } (29)$$

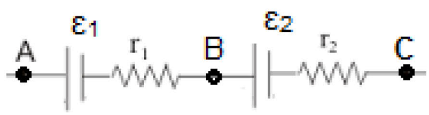
Q. A 1 kW heater is designed to operate on a 200 V supply. Find (a) the resistance of the heater and (b) by what percentage will its power drop if the supply voltage drops to 160 V ?
(36%)

Q. 100 MW of power from a power station is transmitted to a distant substation through a cable of resistance 10 Ω at (a) 20,000 V and (b) 200 V. Find the power loss in the cable in each case. *(2.5×10^8 W, 2.5×10^{12} W)*

- ***Combination of Cells***

a) Cells in Series – Consider two cells of emf $'\varepsilon_1'$ and $'\varepsilon_2'$, having internal resistances $'r_1'$ and $'r_2'$ connected in series. Let $'\varepsilon'$ and $'r'$ be the equivalent e.m.f and internal resistance of the combination respectively. Let I is the current through the cells.

Let V_{AB} is the potential difference between the terminals (A and B) of the first cell, given by,



$$V_{AB} = V_A - V_B$$

$$V_{AB} = \varepsilon_1 - Ir_1 \text{ ----}(30)$$

Similarly,

$$V_{BC} = V_B - V_C$$

$$V_{BC} = \varepsilon_2 - Ir_2 \text{ ----}(31)$$

$$V_{AC} = V_A - V_C$$

$$V_{AC} = (V_A - V_B) + (V_B - V_C)$$

$$V_{AC} = V_{AB} + V_{BC}$$

$$V_{AC} = \varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2$$

$$V_{AC} = (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2) \text{ ---(32)}$$

$$V_{AC} = \varepsilon_{equ} - I\, r_{equ} \text{ ----- (33)}$$

Where, $\varepsilon_{equ} = \varepsilon_1 + \varepsilon_2$ --- (34) and $r_{equ} = r_1 + r_2$ --- (35) are the equivalent emf and internal resistance of the two cells in series respectively.

For 'n' cells in series the equivalent emf and internal resistance are ,

$$\varepsilon_{equ} = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_n \text{ ---- (36) and}$$

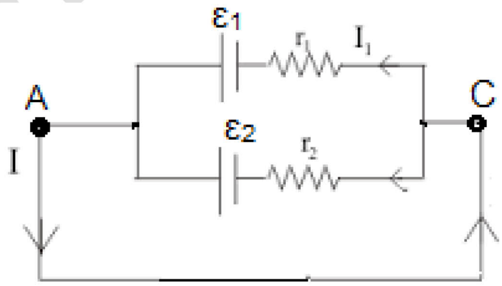
$$r_{equ} = r_1 + r_2 + \cdots + r_n \text{ ---- (37)}$$

If they are identical, each having an e.m.f $'\varepsilon'$ and an internal resistance r, (36) and (37) becomes,

$$\varepsilon_{equ} = n\,\varepsilon \text{ and}$$

$$r_{equ} = n\,r$$

b) Cells in Parallel – Consider two cells of e.m.f $'\varepsilon_1'$ and $'\varepsilon_2'$, having internal resistances $'r_1'$ and $'r_2'$ connected in parallel. Let $'\varepsilon'$ and $'r'$ be the equivalent e.m.f and internal resistance of the combination respectively. Let I_1 and I_2 are the current through the cells.



Since the cells are in parallel, the potential difference across the two cells will be the same. Let V_{AC} is the potential difference between the terminals of the two cells in parallel,

For the first cell,

$$V_{AC} = V = \varepsilon_1 - I_1 r_1$$

$$I_1 = \frac{\varepsilon_1 - V}{r_1} \text{ --- (38)}$$

Similarly for the second cell,

$$V_{AC} = V = \varepsilon_2 - I_2 r_2$$

$$I_2 = \frac{\varepsilon_2 - V}{r_2} \text{ --- (39)}$$

The net current is given by,

$$I = I_1 + I_2$$

Substituting from (38) and (39), the above equation becomes,

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$I = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \text{ --- (40)}$$

Rearranging (40), we get,

$$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \text{ ---(41)}$$

But $V = \varepsilon_{equ} - I\, r_{equ}$ ---- (42)

Comparing (41) and (42), we get,

$$\varepsilon_{equ} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \text{ ---- (43) and}$$

$$r_{equ} = \frac{r_1 r_2}{r_1 + r_2} \text{ ---- (44)}$$

or, $$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \text{ and } \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For 'n' cells in parallel, we have,

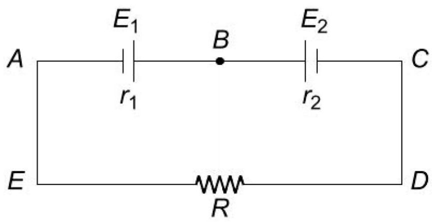
$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \cdots + \frac{\varepsilon_n}{r_n} \text{ ---- (43) and}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_n}$$

If they are identical, each having an e.m.f $'\varepsilon'$ and an internal resistance r, then

$$\varepsilon_{equ} = \varepsilon \text{ and } r_{equ} = \frac{r}{n}$$

Q. Two cells of emfs $E_1 = 1.5\,V$ and $E_2 = 2.0\,V$ and internal resistances $r_1 = 1.0\,\Omega$ and $r_1 = 1.5\,\Omega$ respectively are connected to an external resistor R = 2.5 Ω as shown in figure. Find the potential difference (a) between points A and B (b) between points B and C (c) across R. *(– 1.6 V, 1.85 V, 0.25 V)*

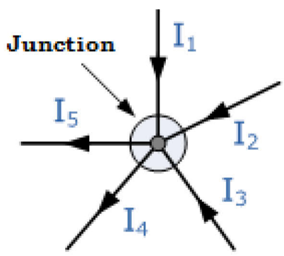


- ***Kirchoff's Rules***
  - ***(a) Kirchoff's Junction Rule (Current Rule or KCL):*** It states that, the algebraic sum of current entering and leaving a junction is equal to zero.

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0 \text{ -- (44)}$$

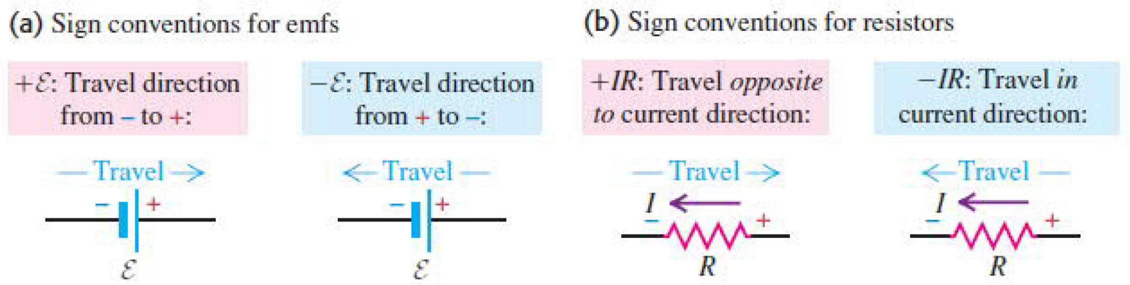
or, $$\sum_{i=1}^{n} I = 0 \text{ --- (44)}$$

or, $$I_1 + I_2 + I_3 = I_4 + I_5$$

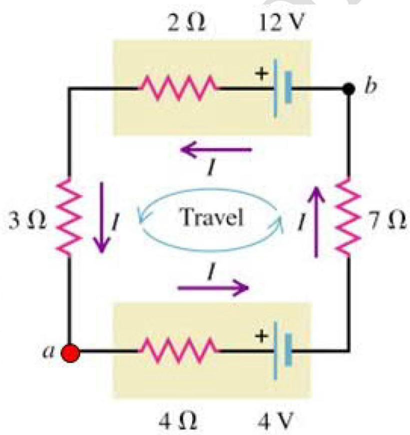


(b) *Kirchoff's Loop Rule (Voltage Rule or KVL):* It states that, the algebraic sum of changes in the potential around any closed loop in an electrical network is always zero.

Sign Convention



Example :



From Second rule , we get,

$$-4I - 4 - 7I + 12 - 2I - 3I = 0$$

Solving, we get, $I = 0.5\ A$

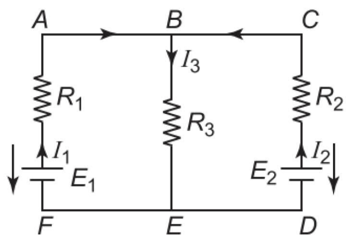
Q. In the circuit shown in Fig., calculate,

(a) the values of currents I_1, I_2 and I_3

(b) the potential difference between points B and E

Given E_1 = 12 V, E_2 = 6 V, R_1 = 5 Ω,

R_2 = 3 Ω and R_3 = 2 Ω



➤ ***Wheatstone Bridge***

A Wheatstone bridge is a network of resistors, used to compare values of unknown resistors and also to find the value of an unknown resistance. Applying KVL, to the loop ADBA, we get,

$$-I_1R_1 + I_gG + I_2R_2 = 0 \quad \text{--- (45)}$$

Applying KVL, to the loop BCDB, we get,

$$-(I_2 - I_g)\,R_4 + (I_1 + I_g)\,R_3 + I_gG = 0 \quad \text{-- (46)}$$

If the current through the galvanometer I_g is zero, the bridge is said to be a balanced Wheatstone bridge.

i.e. if $I_g = 0$, (45) and (46) becomes,

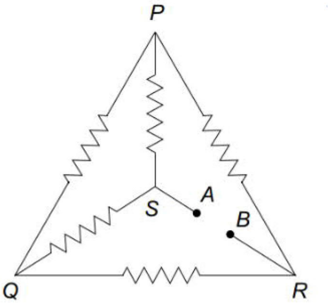
$$-I_1R_1 + I_2R_2 = 0 \quad \text{and}$$

$$-I_2\,R_4 + I_1\,R_3 = 0$$

Dividing the above two equations, we get,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ ---- (47)}$$

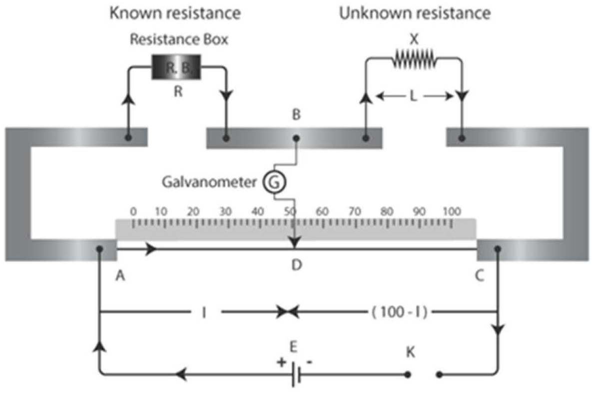
(47) is the condition for a balanced Whetstone bridge.



Q. If each of the resistances shown in the network is R, find the effective resistance between terminals A and B. (Hint: It is a balanced WSB) *(Ans : R)*

Metre Bridge

It is a device used to determine the unknown resistance of a wire. It is based on the principle of a balanced Wheatstone bridge.



Metre bridge

We know that the resistance of a wire is given by, $R = \rho \frac{l}{A}$

For a given wire ρ is a constant. If the area 'A' of the wire is a constant (uniform thickness), then,

$$R \propto l$$

i.e. for a wire of uniform material and cross-section, the resistance is directly proportional to its length.

Working – Close the key 'K' and move the jockey 'J' from A to C, through the meter bridge wire till the galvanometer shows null deflection at a point, say D. Now the bridge is balanced, and the length 'l' is called as the balancing length. Let 'σ' is the resistance per unit length of the meter bridge wire.

Applying the condition for a balanced bridge, we get,

$$\frac{R}{R_{AD}} = \frac{X}{R_{DC}} \text{ ----(48)}$$

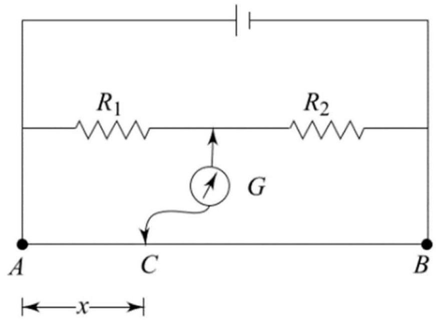
$$\frac{R}{\sigma l} = \frac{X}{\sigma(100-l)}$$

$$X = R\left[\frac{(100-l)}{l}\right] \text{ ----(49)}$$

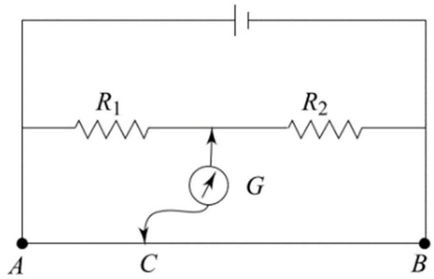
From the known values of R and l, the unknown resistance can be calculated using (49). If the unknown resistance is connected across the left gap and the known resistor (R) across the right gap, (49) becomes,

$$X = R\left[\frac{l}{(100-l)}\right] \text{----(50)}$$

Q. In the meter bridge experiment shown in Fig, the balance length AC corresponding to null deflection is x. What would be the balance length if the radius of the wire AB is doubled? *(Ans : x)*

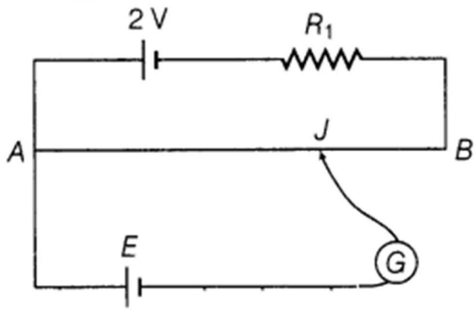


Q. A metre bridge consisting of two resistances R_1 and R_2 connected to a wire AB of length 100 cm. The null point is found to be at a distance 40 cm from end A. When a 12 Ω resistance is connected in parallel with R_2, the null point shifts to 60 cm from A. Find R_1 and R_2. *(Ans : 10 Ω, 15 Ω)*



Potentiometer

It is a device used measure the potential difference between two points. It is used to (a) compare the emf of two cells (b) find the internal resistance of cell.



Principle : The potential difference between two points on a current carrying wire is directly proportional to the length of the wire between the two points, provided, the current through the wire is a constant, the wire has a uniform area of cross-section and made of uniform composition (same material).

From (9) we have,

$$V = I\ \rho\frac{l}{A}$$

If I, A and ρ are constants, we get,

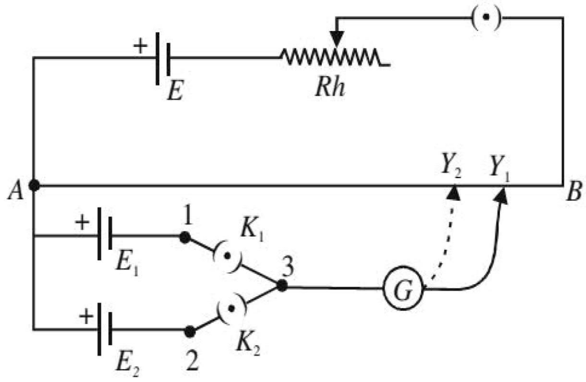
$$V \propto l$$

or, $V = \Phi\, l$ – (51); where Φ is a constant called as the potential gradient.

Applications

(a) To compare the emf of two primary cells

The key is closed and the terminals 1 and 3 of the two-way key is connected so that the cell having the emf ε_1 is introduced to circuit. The jockey is moved from A to B till the galvanometer shows null deflection. Let l_1 is the null point length measured from the end A.



From (51), we get,

$$\varepsilon_1 = \Phi\, l_1 \text{ ----(52)}$$

Now the key is put between the terminals 2 and 3 so that the cell with emf ε_2 is connected to the circuit. Again, the null point length l_2 is measured from the end A of the potentiometer wire.

$$\varepsilon_2 = \Phi\, l_2 \quad \text{---- (53)}$$

Dividing the equations (52) and (53), we get,

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} \quad \text{----(54)}$$

So, if one cell is a standard cell of known emf, the emf of the other cell can be calculated using (54)

(b) To find the internal resistance of a cell.

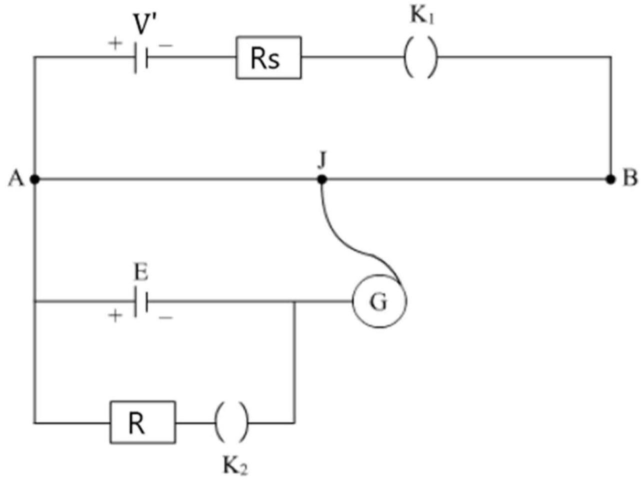
Close the key K_1 and keep the key K_2 open. Move the jockey through the potentiometer wire and locate the null point. Measure the null point length (l_1) from the end A. Now we have,

$$\varepsilon = \Phi\, l_1 \quad \text{---- (55)}$$

Now the key K_2 is closed and again the null point length (l_2) is calculated. We get,

$$V = \Phi\, l_2 \quad \text{-----(56)}$$

Dividing (55) with (56), we get,



$$\frac{\varepsilon}{V} = \frac{l_1}{l_2}$$

But $\varepsilon = V + Ir$

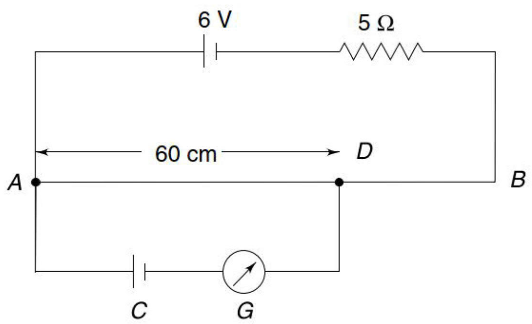
$$\frac{V + Ir}{V} = \frac{l_1}{l_2}$$

$$1 + \frac{Ir}{V} = \frac{l_1}{l_2}$$

$$\frac{Ir}{V} = \frac{l_1}{l_2} - 1$$

$$r = R\left(\frac{l_1}{l_2} - 1\right) \quad \text{---- (57)}$$

Q. In the potentiometer circuit shown in Fig., the internal resistance of the 6 V battery is 1 Ω and the length of the wire AB is 100 cm. When AD = 60 cm, the galvanometer shows no deflection. The emf of cell C is (the resistance of wire AB is 2 Ω) (Ans : 0.9 V)



Q. In the potentiometer circuit shown in Fig., find the value of 'l' when the galvanometer shows no deflection. The length of wire AB is 100 cm and its resistance is 3Ω (Ans: 83.3 cm)