Electric Potential & Capacitance

Charges at Rest / Motion

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CHAPTER 2 : ELECTROSTATIC POTENTIAL & CAPACITANCE

- ***Introduction:*** In the previous chapter we have seen that electric charges creates an electric field and a charge placed in an electric field will experience an electrostatic force. If the test charge is moved through a region of electric field from one point to another, some work is to be done. According to the work – energy principle, this work done is same as the electrostatic potential energy gained by the charge system.

- ***Electrostatic Potential Energy (U)***

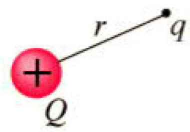

Figure shows a source charge 'Q' which is responsible for the electric field 'E' around it. A test charge 'q' is at a point 'P'. The electrostatic potential energy of the charge 'q' is defined as the work done in bringing the charge from infinity to the point 'P' without any acceleration.

In general, the potential energy of the charge system is same as the work done in building the charge configuration by bringing both charges from infinity to their respective points. The work done to bring the first charge, (say 'Q') is zero since it is moved in the absence of an electric field. Since there is an electric field due to the charge 'Q', some work is to be done against the electrostatic force when the second charge 'q' is brought from infinity to its corresponding point.

The work done by the external force 'F' to move the charge 'q' by a small displacement dr is given by, $dW = \vec{F}.\overrightarrow{dr}$

Integrating, we get, $W = \int_{\infty}^{r} \vec{F}.\overrightarrow{dr}$ ----- (1)

Since the charge is brought without any acceleration, the external force 'F' should have the same magnitude as the electrostatic force 'Fe', but having an opposite direction. i.e. $\vec{F} = -\vec{F_e}$

(1) becomes,

$$W = -\int_{\infty}^{r} \vec{F_e}.\overrightarrow{dr}$$

But, $F_e = K\,\frac{Q\,q}{r^2}$

$$\therefore\ W = -\int_{\infty}^{r} K\,\frac{Q\,q}{r^2}\,dr$$

$$= -K\,Q\,q\int_{\infty}^{r}\frac{1}{r^2}\,dr$$

On integration, we get,

$$W = K\,\frac{Q\,q}{r}$$

By definition, this work is same as the potential energy of the charge 'q'. Therefore, the electrostatic potential energy of the charge 'q' is given by,

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$$U = K\,\frac{Q\,q}{r} \qquad \text{----- (2)}$$

It is a scalar quantity and is measured in *joule* (J)

- ***Electrostatic Potential (V) :*** The electrostatic potential at a point is defined as the work done in bringing a unit positive charge from infinity to that point, without acceleration. It is a scalar quantity and is having SI unit '*volt*' (V).

$$\text{i.e. } V = \frac{W}{q} \quad \text{----- (3)}$$

- ***Electric potential due to a point charge***

Consider a source charge 'Q', responsible for an electric field'E'. 'P' is a point at a distance 'r' from the source charge.

From (3), we have,

$$V = \frac{W}{q}$$

But the work done 'W' is same as the potential energy 'U'

$$\therefore \quad V = \frac{U}{q} \text{ ------ (4)}$$

From, (2) and (4), we get

$$V = K\,\frac{Q}{r} \quad \text{---- (5)}$$

- ***Electric potential due to multiple charges***

Consider a system of charges having charges q_1, q_2, q_3 ....q_n. The electric potential at a point 'P' is the sum of the potentials due to the individual charges.

i.e., the net potential at 'P' is given by,

$V = V_1 + V_2 + V_3 + \ldots + V_n$; where V_1 is the potential at the point 'P' due to charge q_1 and so on.

Q. Two charges 3 × 10⁻⁸ C and –2 × 10⁻⁸ C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. *(9 cm, 45 cm)*

Q. Two charges 2 × 10⁻⁸ C and –2 × 10⁻⁸ C are located 10 cm apart. Calculate the electric potential at the mid-point of the line joining the two charges. *(Zero)*

Q. Six charges, each of value 'q' are placed at the vertices of a regular hexagon of side 'a'. Find the electric field and the electric potential at the centre of the hexagon. Also calculate the work done to move a –5 C charge from the centre to infinity. *(zero, $6\,K\frac{q}{a}$, $30\,K\frac{q}{a}$)*

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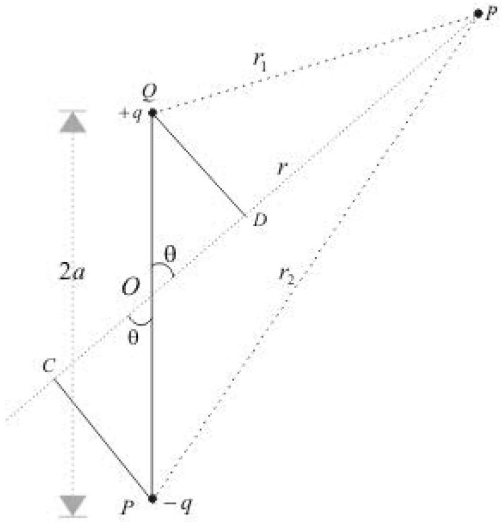
➤ ***Electric potential due to an electric dipole***



Consider an electric dipole of dipole moment '*p*'. '*P*' is a point at a distance '*r*' from the centre of the dipole.

The electric potential at '*P*' is the sum of potentials due to the two charges.

$$V = V_{+q} + V_{-q}$$

$$V = K\frac{q}{r_1} + K\frac{-q}{r_2}$$

$$V = K\,q\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Assuming a point size dipole, i.e. $r >> a$, we have

$$r_1 = r - a\,cos\theta \quad and$$
$$r_2 = r + a\,cos\theta$$

∴ we have $$V = K\,q\left[\frac{1}{r-a\,cos\theta} - \frac{1}{r+a\,cos\theta}\right]$$

Simplifying we get, $$V = K\,q\left[\frac{2a\,cos\theta}{r^2 - a^2cos^2\theta}\right]$$

Since, for a point dipole, $r >> a$, the term $a^2cos^2\theta$ is too small when compared with r^2so that it can be neglected. Also,$2aq = p$, the electric dipole moment.

$$V = K\left[\frac{p\,cos\theta}{r^2}\right] = \frac{1}{4\pi\varepsilon_o}\left[\frac{p\,cos\theta}{r^2}\right] \text{ ---- (6)}$$

In vector form, $$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{\vec{p}\,.\,\hat{r}}{r^2}\right] \text{ ---- (7)}$$

Special Cases :

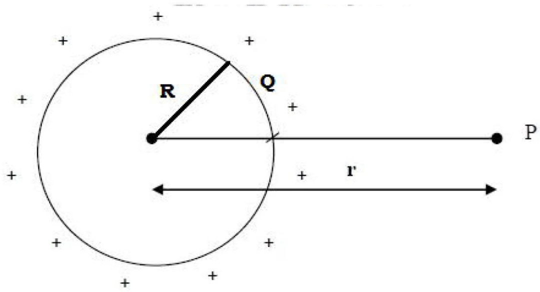
Case 1 : $\theta = 0$; we have $V = \frac{1}{4\pi\varepsilon_o}\left[\frac{p}{r^2}\right]$ (axial point)

Case 2 : $\theta = 180^o$; we have $V = -\frac{1}{4\pi\varepsilon_o}\left[\frac{p}{r^2}\right]$ (axial point)

Case 3 : $\theta = 90^o$; we have $V = 0$ (equatorial point)

➤ ***Electric Potential due to a Uniformly charged Spherical Shell***

Consider a uniformly charged spherical shell of radius 'R', having a surface charge density 'σ'. 'P' is a point at a distance 'r' from the centre of the shell.



The shell behaves as if its charges is concentrated at its centre.

∴ the potential at 'P' is given by,

$$V_P = K\,\frac{Q}{r}$$

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But $Q = \sigma \;\; 4\pi R^2$

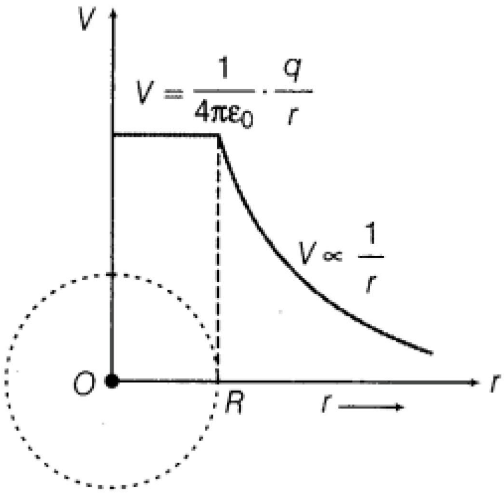
$$\therefore V_P = \frac{\sigma R^2}{\varepsilon_o r} \quad \text{---(8)}$$

On the surface of the shell, $r = R$

$$V_P = \frac{\sigma R}{\varepsilon_o} \quad \text{---- (8)}$$

For a point inside the shell, $r < R$, the potential is same as that on its surface, which is given by (8). This is because, inside the shell, since the electric field is zero, there is no extra work needed to move the charge from the surface to a point inside the shell.

Graphical representation of potential verses distance from the centre



- ***Equipotential Surfaces***

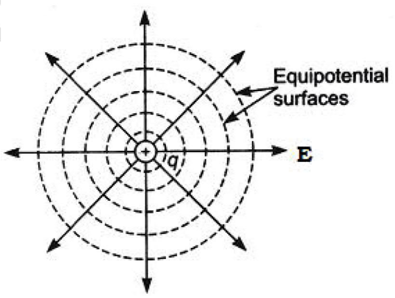
An equipotential surface is a surface where the potential remains a constant.

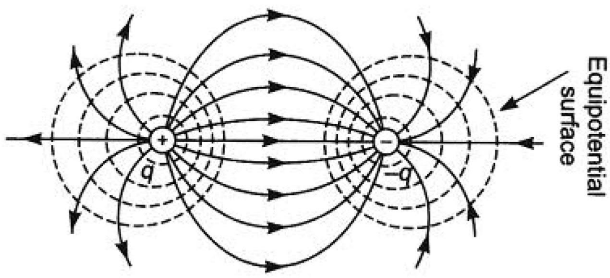
Some properties of an equipotential surface are :

a) The work done to move a charge from one point to other on the equipotential surface is zero.

b) The electric field lines are always perpendicular to the equipotential surface.

c) Two equi-potential surfaces will never intersect.





Equipotential surfaces due to electric dipole

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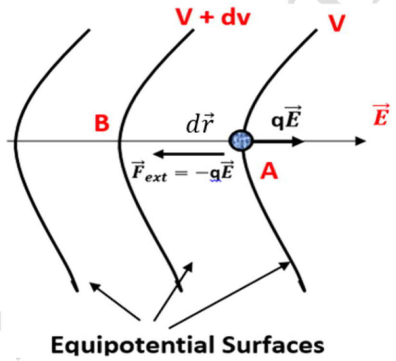
Q. (a) A point 'P' is 20m away from a 2uC point charge and 40 cm away from a 4uC point charge. Find the electric potential at 'P'.
(b) Find the work done to bring a 0.2 C charge from infinite distance to the point 'P'.
(c) Find the work done to bring -0.4C charge from the point 'P' to infinity ?
(1800V, 360J, 720J)

➢ ***Relation between electric field (E) and electric potential (V)***

Consider two equipotential surfaces A and B, having potentials V and $V + dV$ respectively. A charge 'q' is moved from A to B and let dW is the work done in the process, which is given by,

$$dW = q\,dV \text{ ---- (1)}$$

The work done is also given by,



$$dW = \vec{F}.\vec{dr}$$

$$dW = -F\,dr \text{ ----(2)}$$

$(\theta = 180^0)$

Equating (1) and (2), we get,

$$-F\,dr = q\,dV$$

or

$$\frac{F}{q} = -\frac{dV}{dr}$$

$$E = -\frac{dV}{dr} \text{ ----- (3)}$$

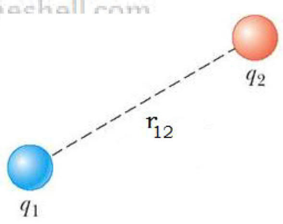
Which means that the electric field 'E' points in the direction opposite to the direction of increasing potential 'V'. In other words,

a) Electric field is in the direction in which the potential decreases the steepest.
b) The magnitude of electric field is the magnitude of potential per unit displacement normal to the equipotential surface at a point.

➢ ***Potential Energy of a system of charges***

The potential energy of a system of charges is defined as the work done to build the configuration of charges. This means that we consider all the charges are initially at infinity and they are brought one after other from infinity to their respective positions. The total work done by the external agency in this process is defined as the electrostatic potential energy of the system.

Consider a two-charge system, having charges q_1 and q_2, as shown in the figure. The work done to bring the first charge q_1 from infinity to its position is zero since



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there is no external electric field against work is to be done.

$$\text{i.e. } W_1 = 0$$

The work done to bring the second charge q_2 from infinity to its respective position is given by,

$$W_2 = q_2 V \text{ ; where } V \text{ is the potential created by } q_1.$$

$$W_2 = q_2 K \frac{q_1}{r_{12}} = K \frac{q_1 q_2}{r_{12}}$$

∴ the total work done is given by,

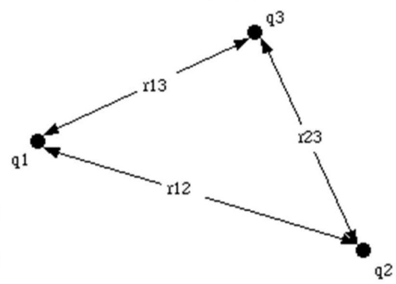
$$W = W_1 + W_2$$

$$W = K \frac{q_1 q_2}{r_{12}}$$

The work done (W) is the potential energy (U)of the system,

$$\therefore \; U = K \frac{q_1 q_2}{r_{12}} \text{ ---- (4)}$$

Similarly, for a three-charge system, the work done to bring the three charges q_1, q_2 and q_3 are respectively,



$$W_1 = 0$$

$$W_2 = K \frac{q_1 q_2}{r_{12}} \quad \text{and}$$

$$W_3 = K \left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

The P.E. of the system is therefore $U = W = W_1 + W_2 + W_3$

$$U = K \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right] \text{ ------ (5)}$$

Generalising the energy for a system having 'n' charges, we have,

$$U = K \sum_{\substack{i,j=1 \\ i<j}}^{n} \frac{q_i q_j}{r_{ij}}$$

- ***Potential Energy of a system of charges in an external electric field***

Consider a two charge system, having charges q_1 and q_2, placed in a region of electric field of intensity 'E'. The work done to bring the first charge q_1 from infinity to its position is given by,

$W_1 = q_1 V(r_1)$, where $V(r_1)$ is the potential at the location of q_1 due the external electric field.

The work done to bring the second charge q_2 from infinity to its respective position is given by,

$$W_2 = q_2 V(r_2) + K \frac{q_1 q_2}{r_{12}}$$

Where the first term on the RHS is the work done against the eternal electric field and the second term is the work done against the electric field produced by q_1.

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∴ the total work done which is same as the potential energy of the system, is given by,

$$U = q_1 V(r_1) + q_2 V(r_2) + K \frac{q_1 q_2}{r_{12}}$$

For a system having 'n' charges, in an external electric field,

$$U = q_1 V(r_1) q_2 V(r_2) + \cdots + q_n V(r_n) + K \sum_{\substack{i,j=1 \\ i<j}}^{n} \frac{q_i q_j}{r_{ij}}$$

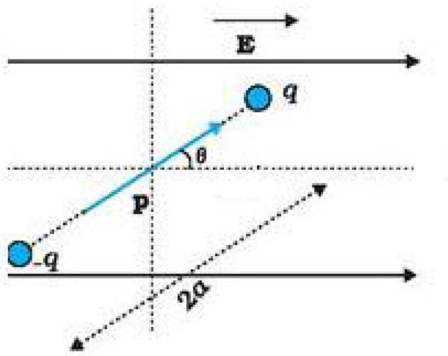
Q.(a)Determine the electrostatic potential energy of a system consisting of two charges 7 μC and –2 μC (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external electric field E = A (1/r [2]); A = 9 × 10[5] C m[-2]. What would the electrostatic energy of the configuration be? *(–0.7 J, 0.7 J., 49.3 J)*

- ***Potential Energy of a dipole in a uniform electric field*** $\vec{E}$

Consider an electric dipole having a dipole moment '*p*' placed in an external uniform electric field of intensity '*E*'. Let θ is the angle between the '*p*' and '*E*'.



Let dW is the work done to rotate the dipole through a small angle $d\theta$.

i.e. $dW = \tau \, d\theta$

$dW = p\,E\,\sin\theta \, d\theta \quad (\tau = pE\sin\theta)$

the net work done $W = \int_{\theta_i}^{\theta_f} p\,E \sin\theta \, d\theta$

$$W = pE \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$$

$$W = -pE \left[\cos\theta_f - \cos\theta_i\right]$$

The work done is same as the energy gained by the system,

$$\Delta U = W$$

$$\Delta U = -pE \left[\cos\theta_f - \cos\theta_i\right] \text{ --- (6)}$$

We consider* the potential energy of the dipole to be zero at an angle $= 90^o$. Let $\theta_i = 90^o$ and $\theta_f = \theta$.

$$\Delta U = U = -pE \left[\cos\theta - \cos 90\right]$$

$$U = -pE \cos\theta \text{ ---- (7)}$$

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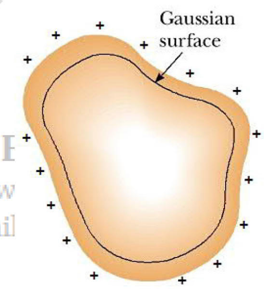
To recapitulate, for a dipole in a uniform electric field,

Angle (θ)	*Net Force* (F)	*Net Torque* (τ)	*Potential Energy*	*Known as*
0	0	0	$-pE$	*Stable Equlibrium*
90^o	0	PE	0	–
180^o	0	0	pE	*Unstable Equlibrium*

- ***Electrostatics of Conductors***

a) *The excess charges given to a conductor always reside on its surface.* This is due to the mutual repulsion between the charges so that they would stay at the maximum possible distance apart, which is the surface of the conductor.

b) *The electric field inside a charged conductor is zero at static conditions.* Consider a Gaussian surface just below the surface of the charged conductor. From Gauss' law, we have,



$$\oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

But the charge enclosed by the Gaussian surface (q) is zero, which implies that the electric field inside the conductor is zero.

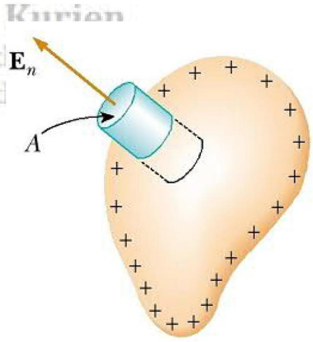
c) *The electric field is normal to the surface of a charged conductor.* If not, there will be a component of electric field parallel to the surface of the conductor which causes the charges to drift which never happens in static situations.

d) *The electric potential inside a charged conductor is a constant which is same as the potential on its surface.*
We know that,

$$E = -\frac{dV}{dr}$$

Since the electric field inside the conductor is zero, the potential must be a constant.
(In other words, there is no extra work to be done to move a charge from the surface to a point inside the conductor due to the absence of electric field inside the conductor.)

e) *The electric field on (or just outside) the surface of a charged conductor has a magnitude equal to* $\frac{\sigma}{\varepsilon_0}$



Consider a point (P) just outside the charged conductor and a cylindrical gaussian surface

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partly inside and partly outside, having the point (*P*) on its base, as shown in the figure. The flux through the cylinder lying inside the conductor is zero since the electric field inside the conductor is zero. The flux through the curved surface of the cylinder outside the conductor is zero as the electric field makes an angle 90 with the area vector. There is a flux only through the base which lies outside the conductor.

From Gauss' law, we have,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

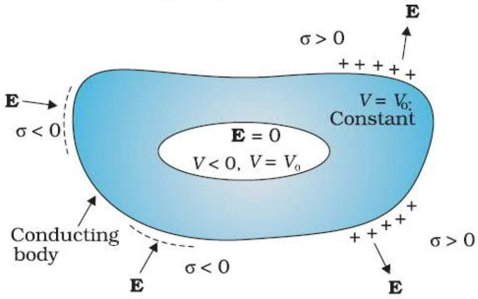
$$\oint E\, ds = \frac{q}{\varepsilon_0} \quad (\theta = 0)$$

$$E \oint ds = \frac{q}{\varepsilon_0} \qquad (8)$$

$$E\, A = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

f) Electric field inside a cavity in a conductor of any shape or size is zero, provided no charge resides inside the cavity. This property of the conductor is known as *electrostatic shielding.*

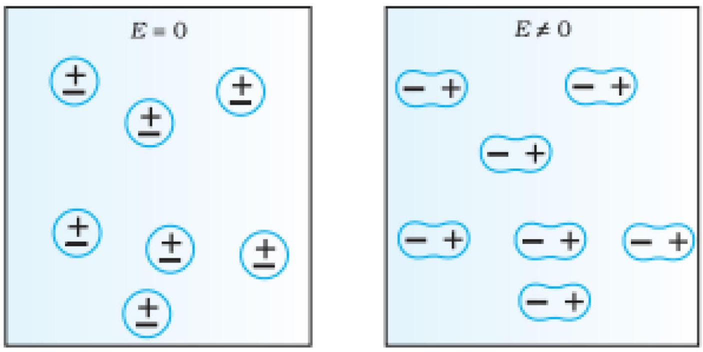


➢ **Dielectrics**

Dielectrics are basically insulators. The two types of dielectrics are :

i. Non –polar dielectric
ii. Polar dielectric

For a non – polar dielectric molecule, the centres of the effective positive and negative charges coincide and hence they possess zero net dipole moment. A non- polar dielectric slab hence possesses zero permanent dipole moment. When the slab is place in an external electric field, will possess an induced dipole moment. This is due to the stretching of the molecule since the positive and



(a) Non-polar molecules

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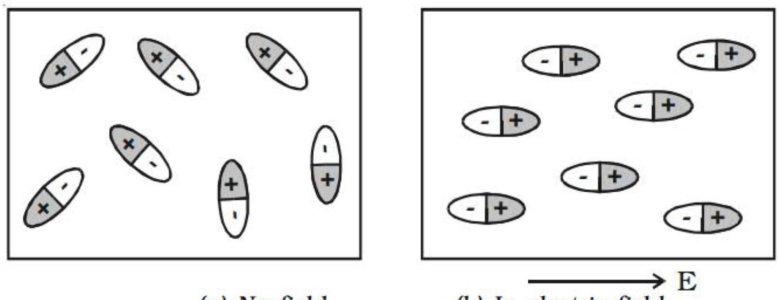
negative charges moves in opposite direction in the presence of san electric field and the dielectric is said to be polarised.

In the case of a polar dielectric molecule, the centres of the effective positive and negative charges do not coincide and hence they possess a permanent dipole moment. But, a dielectric slab of polar molecules will possess zero net dipole moment due to the random arrangement of molecules. When it is placed in an electric field, the molecules will experience a torque and gets aligned in the direction of the external electric field so that the slab and they will possess an induced dipole moment.

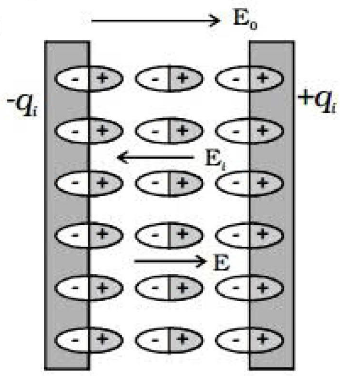


(a) No field *(b) In electric field*

Polar molecules

Due to the polarisation of the dialectic slab, an induced electric field E_i is developed inside the dielectric, having a direction opposite to the external electric field E_o.Therefore the net electric field inside the dielectric slab decreases, which is given by,

$$E = \frac{E_o}{K} \text{ where K is the dielectric constant.}$$



Polarisation of dielectric material

The net dipole moment per unit volume is called as the polarisation. It is a vector quantity having an SI unit C/m².

$$\vec{P} = \frac{\vec{p}_{net}}{V}$$

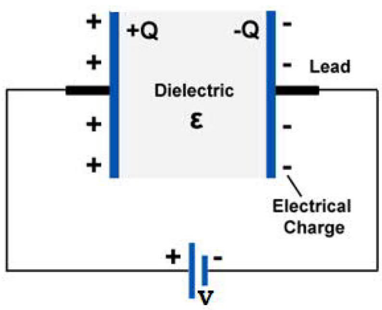
It is observed that the polarisation P is directly proportional to the external electric field E_o

i.e. $P \alpha E_o$

or, $P = \chi E_o$, where χ is a constant called as the 'electric susceptibility'. It is a measure of the extent to which a dielectric gets polarised.

- ***Capacitors***

A capacitor is an arrangement of two conductors separated by a dielectric. It is used to store electric charge and electrical energy.



When the two conductors are connected across a potential difference, electrons are

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transferred from one conductor to the other so that one conductor gets a positive charge and the other an equal negative charge. Now the capacitor is said to be charged. The capacitor continues to get charged till the potential difference across the capacitor is same as that across the cell. The net charge on the capacitor is zero.

It is easy to see that the charge on the capacitor 'Q' is directly proportional to the applied potential difference 'V'.

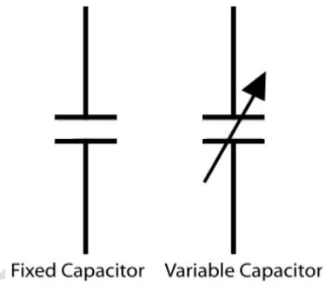
i.e., $Q \;\alpha\; V$

$\therefore Q = C\,V$ ---(9); where C is a constant known as the capacitance of the capacitor.

Capacitance is the measure of the ability of the capacitor to store charge. The SI unit of capacitance is farad (F). Since 1 farad is a very large quantity, smaller units such as μF, nF and pF are commonly used.

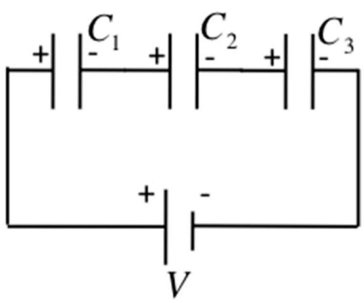
Capacitance of a capacitor depends on the size, shape, separation of the two conductors & the nature of the dielectric separating the two conductors.

Capacitor Symbol



- ***Combination of Capacitors***

a) Capacitors in Series: Consider three capacitors C_1 C_2 and C_3, connected in series and the combination connected across a cell of potential difference 'V', as shown in the figure.



In a series combination, the charge on each capacitor will be the same whereas the applied potential difference 'V' will divide across each capacitor.

Let Q is the charge on each capacitor and C is the equivalent capacitance of the combination.

The potential difference can be written as,

$$V = V_1 + V_2 + V_3 \text{ --- (1);}$$

where V_1 is the potential difference across C_1 and so on.

But $V = \frac{Q}{C}$

$\therefore$ (1) becomes,

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$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{--- (2)}$$

If there are 'n' capacitors in series, the equivalent capacitance is given by,

$$\therefore \ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} \quad \text{--- (3)}$$

If there are 'n' identical capacitors of capacitance C in series, the equivalent capacitance is given by,

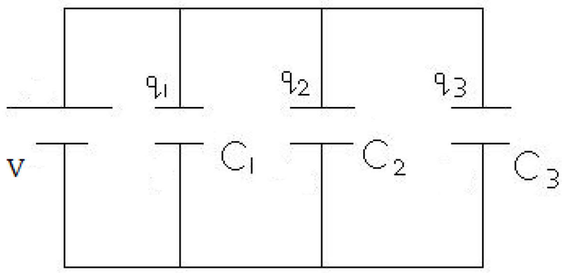
$$C_{eq} = \frac{C}{n} \text{ ----- (4)}$$

b) Capacitors in Parallel : Consider three capacitors C_1 C_2 and C_3, connected in parallel and the combination connected across a cell of potential difference 'V', as shown in the figure. *In a parallel combination, the potential difference across each capacitor will be the same whereas the total charge 'Q' will divide between each capacitor.*

Let C is the equivalent capacitance of the combination. The net charge 'Q' can be written as,

$$Q = Q_1 + Q_2 + Q_3 \quad \text{--- (1)} \quad \text{But } Q = C\,V$$

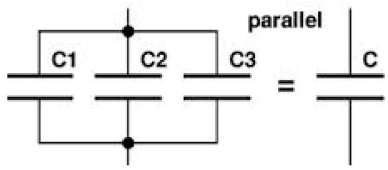
$\therefore$ (1) becomes,



$$CV = C_1 V + C_2 V + C_3 V$$

$$C = C_1 + C_2 + C_3 \quad \text{----- (2)}$$

If there are 'n' capacitors in parallel, the equivalent capacitance is given by,



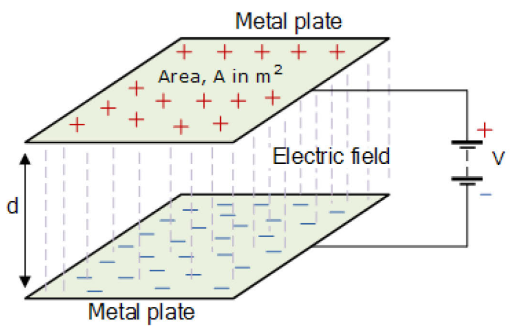
$$C = C_1 + C_2 + C_3 + \cdots + C_n \quad \text{--- (3)}$$

If there are 'n' identical capacitors of capacitance C in parallel, the equivalent capacitance is given by,

$$C_{eq} = n\,C \quad \text{----- (4)}$$

- ***Parallel Plate Capacitor***

A parallel plate capacitor consists of two metal plates, having an area 'A', placed parallel to each other at a distance 'd' apart in vacuum (or air).



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When the plates of the capacitor are connected across a potential difference, it gets charged and an electric field 'E' is established between the plates. Since the electric field between the plates is uniform, the potential difference 'V' can be expressed as,

$$V = E\, d \text{ --- (1)}$$

The electric field between the plates of the capacitor is given by,

$$E = \frac{\sigma}{\varepsilon_o}$$

But $\sigma = \frac{Q}{A}$ $\qquad \therefore\ V = \frac{Q\, d}{A\, \varepsilon_o}$

$$\frac{Q}{V} = \frac{\varepsilon_o\, A}{d}$$

$$\therefore C = \frac{\varepsilon_o\, A}{d} \quad \text{----- (2)}$$

If there is a dielectric of dielectric constant 'K' completely occupying the space between the plates, (1) becomes,

$$V = \frac{E}{K} d \quad \text{--- (3)}$$

Proceeding further as in the previous case, we get the capacitance of the capacitor C' as

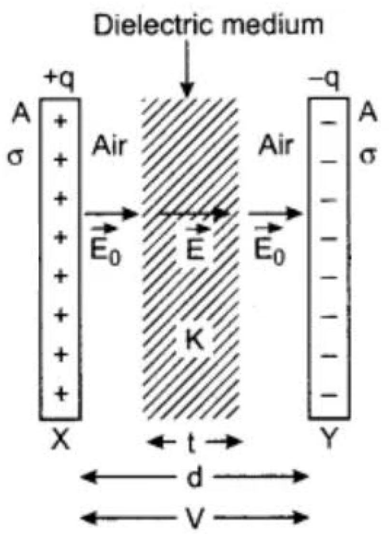
$$C' = K \frac{\varepsilon_o\, A}{d} \text{ ----- (4)}$$

From (2) and (4), we get,

$$C' = K\, C \text{ ----- (5)}$$

- ***Effect of dielectric on capacitance of a parallel plate capacitor***

Consider a parallel plate capacitor having a capacitance C. A dielectric slab of dielectric constant 'K' and thickness 't' ($t < d$) is introduced between the plates of the capacitor so that the capacitance becomes C'.



The capacitor is connected across a potential difference 'V', so that electric fields 'E_o' and $\frac{E_o}{K}$ exists between the plates, outside and inside the slab respectively. The potential difference 'V' can be expressed as,

$$V = E_o(d - t) + \frac{E_o}{K} t$$

$$V = E_o \left[d - t + \frac{t}{K} \right] \text{--- (6)}$$

But $E_o = \frac{\sigma}{\varepsilon_o} = \frac{Q}{A\, \varepsilon_o}$

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$\therefore$ (6) becomes, $V = \frac{Q}{A\,\varepsilon_0}\left[d - t + \frac{t}{K}\right]$

$$\frac{Q}{V} = \frac{\varepsilon_0 A}{\left[d - t + \frac{t}{K}\right]}$$

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)} \quad \text{---- (7)}$$

Case 1 : Let $K = 1$ (vacuum), (7) becomes

$$C = \frac{\varepsilon_0 A}{d} = C_0 \text{ , which is (2).}$$

Case 2 : Let $K = \infty$ (conducting slab), (7) becomes

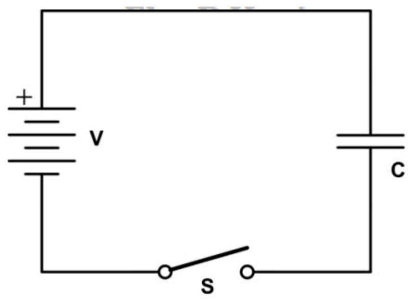
$$C = \frac{\varepsilon_0 A}{d - t}$$

Case 3 : Let $t = d$ (dielectric is completely filled), (7) becomes,

$$C = K\frac{\varepsilon_0 A}{d} \text{ , which is (4).}$$

- ***Energy Stored by a Capacitor***

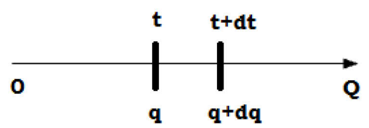
Consider a capacitor of capacitance 'C', initially uncharged, connected across a cell of potential difference 'V' through a switch 'S'. When the switch is closed the capacitor gets charged to a value 'Q'. Let 'q' is the charge on the capacitor at an instant. The work done by the cell in charging the capacitor is same as the energy stored by the capacitor.



Let dW is the work done by the cell to charge the capacitor by an amount $'dq'$, which is given by,

$$dW = V'dq$$

$$dW = \frac{q}{C}\,dq \quad \text{--- (1)}$$



The total work done by the cell to charge the capacitor to a value 'Q' is obtained by integrating (1)

$$W = \int_0^Q \frac{q}{C}\,dq \quad \text{--- (2)}$$

On integrating (2), we get ,

$$W = \frac{Q^2}{2C} \quad \text{--- (3)}$$

Since the work done by cell given (3) is same as the energy stored by the capacitor, we get,

$$U = \frac{Q^2}{2C} \quad \text{------- (4)}$$

or $$U = \frac{1}{2}C\,V^2 \quad \text{--- (5)}$$

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or $U = \frac{1}{2} Q\, V$ ---- (6)

Energy Density (u) : It is defined as the energy stored by the capacitor per unit volume. From (5) we have, $U = \frac{1}{2} C\, V^2$

$$\text{But } C = \frac{\varepsilon_o A}{d} \quad \text{and } V = E\, d$$

$$\therefore U = \frac{1}{2} \frac{\varepsilon_o A}{d} E^2 d^2$$

$$U = \frac{1}{2} \varepsilon_o A\, E^2\, d$$

$$\frac{U}{A\, d} = \frac{1}{2} \varepsilon_o E^2$$

The product $A\, d$ is the volume of the capacitor. Therefore the LHS of the equation represents the energy density (u)

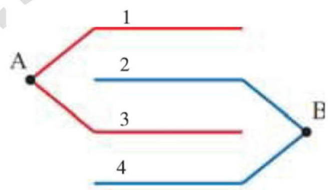
$$\therefore u = \frac{1}{2} \varepsilon_o E^2 \quad \text{------ (7)}$$

Q.A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness (3/4) d, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates? $\left(\frac{4K}{K+3} C\right)$

Q.A Capacitor of 4uF is charged to 50V. It is then connected across a 2uF uncharged capacitor. Calculate the total energy of the above system.

(3.3 x 10^{-6} J)

Q. In the figure, area of each plate is A and the distance between the consecutive plates is 'd'. What is the effective capacitance between the points A and B? $\left(3 \frac{\varepsilon_o A}{d}\right)$



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