Electric Charges & Fields

Charges at Rest

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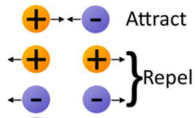
CHAPTER 1 : ELECTRIC CHARGES & FIELDS

Introduction

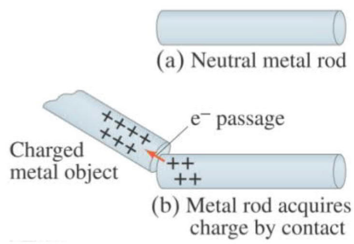
Ancient Greeks observed that when amber and fur rubbed together, amber acquired a property to attract certain objects.
When an object acquires the property of amber to attract other objects, it is said to be (electrically) charged.



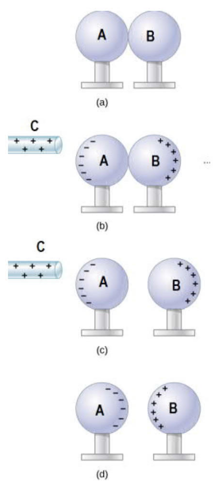
Electrostatics is a branch of physics which deals with electric **charges at rest**.
From experiments it was found that there are two types of charges, positive and negative. Like charges repel whereas unlike charges attract.
A body acquires a positive when electrons are removed (lost) from the body. Similarly, a body acquires a negative charge when it gains electrons. In other words, a body gets charged when **electrons** are removed or added to the body. A charged body can attract an uncharged body by inducing charges on it. Hence only repulsion (not attraction) shows that the two given bodies are charged.
Electric charge is a scalar quantity. The SI unit of charge is coulomb (C) and its dimensions are [AT]

Methods of charging a body

The different ways by which we can charge a body are :
1. Charging by friction : When we rub two bodies, electrons gets transferred from one body to the other. The body which loses electrons acquires a positive charge, the other, which gains electrons, acquires a negative charge. Since the negatively charged body gains electrons, there is a microscopic increase in the mass of the body.
2. Charging by conduction : In this method of charging, the uncharged body, which is to be charged, is put in contact with a charged body. The uncharged body acquires the same type of charge as that of the charged body. In fig, a neutral body (rod) is put in contact with a positively charged body. The positive charges are transferred to the uncharged body. In reality, the electrons are transferred from the uncharged body to the charged body.



3. Charging by induction: In this method of charging, the uncharged body, which is to be charged, is brought near to a charged body.
Figure shows the different steps on charging two conducting bodies. Initially the two bodies A and B are kept in contact and then a charged body C is brought near it, which causes charge separation on the two bodies A and B. Keeping the body C, the bodies A and B are separated. Now the body C is removes which results in two oppositely charged bodies A and B.



The device used to detect the presence and type of charge is called an **electroscope**, or more specifically called as the gold

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leaf electroscope. An electroscope consists of thin conducting foils or leaves. When the electroscope is charged, the leaves acquire the same kind of charge and repel each other, which cause the leaves to diverge. The divergence of the leaves shows the presence of charge.

Why gold foil ?
Gold is having the highest malleability, so it can be made into a very thin foil, which make it extremely light weight.

Properties of Charges

1. Additive property of Charge: If q_1, q_2, q_3 …. q_n are 'n' charges present in a body, then the net charge on a body is the algebraic sum of individual charges.

i.e. net charge $Q = q_1 + q_2 + q_3 + \ldots + q_n$ Or [$Q = \sum_{i=1}^{n} q_i$]

For e.g. if a body consists of three charges, 4C, -2C and 6C, the net charge on the body

$$Q = 4 + (-2) + 6C = 8C$$

2. Conservation of electric Charge : For an isolated system, the net charge is always conserved. It should be noted that charges may be created or be destroyed, but the net charge on the system remains the same.

Take the case of an electron – positron (matter –antimatter) annihilation. i.e.

$$e^- + e^+ \rightarrow \gamma$$

Net charge on both sides is same equal to zero. [Charge of a photon (γ) is zero]

3. Quantisation of electric charge: A physical quantity is said to be quantized if it can take only some discrete values. It is found that the charge on a body is always an integral multiple of a fundamental value.

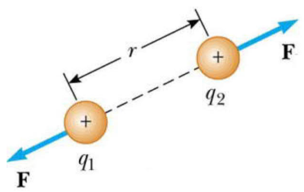
i.e. $q = ne$ where n is an integer (0, ±1, ±2 …) and $e = 1.6 \times 10^{-19}$ C – basic amount of charge, first measured by Robert Millikan (oil drop experiment)

It means that, a body can have a charge of 2e, 5e, -7e etc, but not 2.8e or 3.1e.

Q. How many electrons should be removed from a body so that it acquires a charge of 1C ? *(6.25×10^{18})*

Coulomb's Law

The law states that, the magnitude of force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.



Mathematically,

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$$F \propto \frac{q_1 q_2}{r^2}$$

(Point charges are charges whose dimensions are negligible compared to other dimensions, i.e., size of the charge $<< r$)

Properties of electrostatic force

1. Unlike gravitational force, electrostatic force can be attractive or repulsive
2. It is a long range force
3. It is much stronger than the gravitational force
4. It is a conservative force. i.e. the work done by this force is independent of the path taken.
5. It is a central force as it acts along the line joining the two charges.

From Coulomb's law, $F \propto \frac{q_1 q_2}{r^2}$

$$F = K\frac{q_1 q_2}{r^2}$$ ------ (1) where K is a constant

$K = \frac{1}{4\pi\varepsilon_o}$ = $9 \times 10^9\ Nm^2C^{-2}$; where ε_o is the permittivity of free space

$\varepsilon_o = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

$$K = \frac{1}{4\pi\varepsilon_o}$$

$K = 9 \times 10^9\ Nm^2C^{-2}$

$\varepsilon_o = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

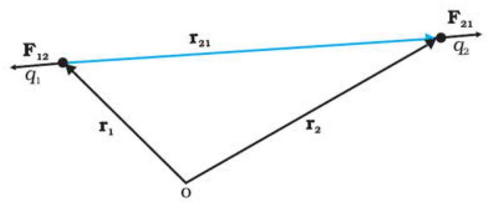
Q. If the distance is increased twice, what is the effect on the force?

(become ¼ times)

Coulomb's Law in vector form :

Equation (1) of Coulomb's law gives only the magnitude of force between two point charges. Since force is a vector quantity, it has both magnitude and direction and hence it is necessary to express the electrostatic force in vector form.

Consider two-point charges q_1 and q_2 separated by some distance $|\vec{r}_{21}|$. Since both the charges are positive, they repel each other.



$\vec{r}_{21}$ is a vector pointed from q_1 to q_2 and $\vec{r}_{12}$ is a vector pointed from q_2 to q_1

$\therefore \quad \vec{r}_{21} = -\vec{r}_{12}$, but , $|\vec{r}_{21}| = |\vec{r}_{12}| = r_{12} = r_{21}$

The force on the charge q_2 due to q_1,

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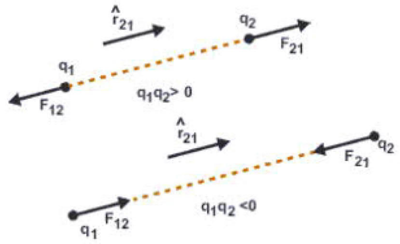
$$\vec{F}_{21} = K \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} \text{ ---------- } (2)$$

And the force on the charge q_1 due to q_2,

$$\vec{F}_{12} = K \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \text{ ---------- } (3)$$

But, $\vec{r}_{21} = -\vec{r}_{12}$ and hence $\vec{F}_{21} = -\vec{F}_{12}$ ------ (4)

Eq (2) shows that the force $\vec{F}_{21}$ is in the direction of $\vec{r}_{21}$ and Eq (3) shows that the force $\vec{F}_{12}$ is in the direction of $\vec{r}_{12}$ i.e. opposite to $\vec{r}_{21}$



Now, consider q_2 is negative so that they attract each other. Equation (2) becomes,

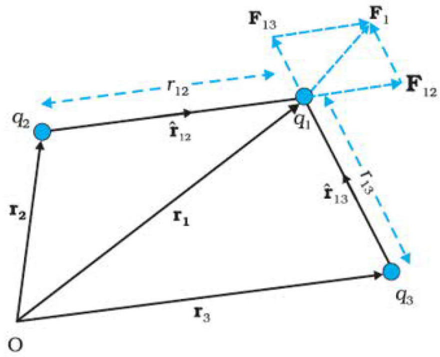
$$\vec{F}_{21} = -K \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} \text{ ---------- } (2),$$

which shows that the force on q_2 due to q_1 is having a direction opposite to $\vec{r}_{21}$ i.e., the charge q_2 gets attracted by q_1 and so on.

In short, if $q_1 q_2 > 0$ $\vec{F}_{21} \,||\, \vec{r}_{21}$ and $\vec{F}_{12} \,||\, \vec{r}_{12}$ (both positive or both negative, repulsion) and if $q_1 q_2 < 0$ $\vec{F}_{21} \,||\, \vec{r}_{12}$ and $\vec{F}_{12} \,||\, \vec{r}_{21}$ (any one charge negative, attraction)

<u>Force due to multiple charges :</u> In the previous topics we had only two point charges. What if we have a system of more than two charges?

Consider a system with three charges q_1, q_2 and q_3. The net force acting on any charge can be found by taking the vector sum of all the forces acting on the charge due to remaining charges. This is called the principle of superposition.



For the system shown in the figure, the net force on the charge q_1 can be calculated by taking the vector sum of the two forces, one due to the charge q_2 and the other due to the charge q_3.

i.e. the net force on the charge q_1,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$

Now, generalizing, if there are 'n' charges, q_1, q_2, q_3 … q_n, then the net force on q_1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots + \vec{F}_{1n}$$

Q. Three charges particles are arranged in a line. Calculate the net electrostatic force on charge q_1 due to the other two charges. (Ans : 5.7N, towards right)

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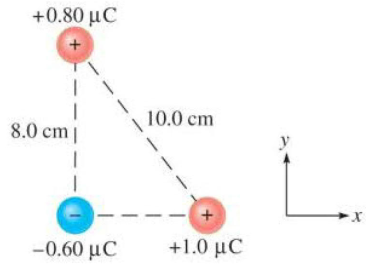
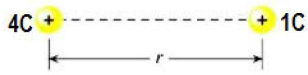
Q. Three-point charges are fixed in place in a right triangle. What is the electrostatic force on the –0.60 μC charge due to the other two charges? (Ans: $1.50\,\hat{\imath} + 0.67\,\hat{\jmath}$)



Q. Two charges 4C and 1C are separated by a distance 'r' as shown in the figure.



a) Where should a charge q be placed on the line joining the two charges so that the charge q is in equilibrium ?
b) What should be the value of q, so that the other charges are in equilibrium?
c) Is the charge q in stable or unstable equilibrium ?

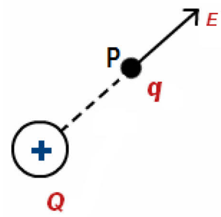
(Ans : a) between the two charges, at a distance $\frac{2}{3}$ r from the 4C charge

b) $q = -\frac{4}{9}$ C

c) Unstable equilibrium

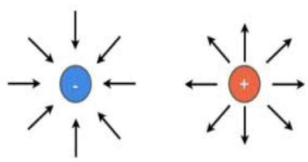
Electrostatic field : The electric field at a point is defined as the electrostatic force acting on a unit positive charge at that point. It's a vector quantity, having an SI unit N/C or V/m

Consider a source charge Q (source of electric field) and a point P. To find the electric field at the point P, assume we place a positive test charge q at that point. Now the electric field at P is given by,



$$\vec{E} = \frac{\vec{F}}{q} \text{ ------------- (5)}$$

The electric field and the electrostatic force are in same direction. For a positive source charge, the electric field is radially outwards whereas for a negative source charge it is radially inwards. We assume that the test charge q is very small so that it doesn't disturb the source charge Q.



We know that, from Coulombs law, the force between the two charges Q and q is

$$\vec{F} = K\,\frac{Qq}{r^2}\,\hat{r}$$

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Substituting this value in (5), we get,

$$\vec{E} = K\,\frac{Q}{r^2}\,\hat{r} \text{ -------- (6)}$$

Also, from (5)

$$\vec{F} = q\vec{E} \text{ --------- (7)}$$

Points to be noted:

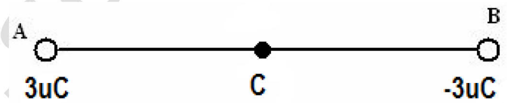
1. If the charge Q is positive (Q>0), $\vec{E}$ is directed radially outwards
2. If the charge Q is negative (Q<0), $\vec{E}$ is directed radially inwards
3. The electric field due to a point charge exhibits spherical symmetry
4. A charge placed in an electric field will experience a force given by (7)
5. Positive charges (e.g. proton) always move in a direction same as that of the electric field.
6. Negative charges (e.g. electron) always move in a direction opposite to that of the electric field.
7. Points where the net electric field is zero are called null points.

Electric field at a point due to multiple charges: Consider a system with 'n' charges q_1, q_2 … q_n. The net electric field at any point can be found by taking the vector sum of electric field due to individual charges at that point. I.e. applying the principle of superposition.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n \text{ -------------(8)}$$

Q. Two-point charges 3μC and -3μC are placed 20cm apart. Calculate the electric field intensity at the midpoint C. If a negative charge of magnitude 1.5 x 10^{4} C is placed at C, what is the force acting on it ?

(5.4 x 10^{6} N/C, 8.1 x 10^{-3} N/C)

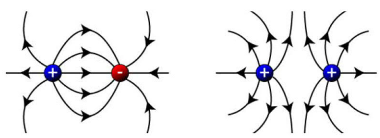


Q. Six charges each of value 'q' is placed at the vertices of a regular hexagon. What is the electric field at the centre of the hexagon? (Zero)

Electric field Lines: Electric field lines are imaginary curves in space such that the tangent at every point on the curve gives the direction of electric field at that point.

Properties of electric filed lines:

1. Electric field lines start from positive charge and ends at negative charge. For a single point charge, electric field lines start or end at infinity.



2. Electric field lines never form a closed loop.

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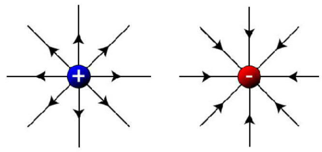
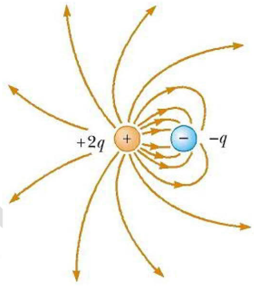
3. In a charge free region, electric field lines are continuous curves, without any breaks.
4. Tangent at any point on the electric field line gives the direction of electric field at that point.
5. Two electric field lines will never intersect each other.
6. The closeness of electric field lines represents the strength of electric field. More closely the electric field lines are, stronger *is the electric field.*



(We can draw infinite number of electric field lines from a charge.But if the charges are having different magnitude, then electric field lines starting and ending should be drawn according to their magnitude of charges)



e.g. Draw electric field lines between two charges +2q and –q.

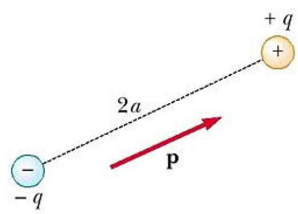
Note that the number of electric field lines originating from the +2q charge is twice that of electric filed lines terminating on the –q charge.

Electric Dipole: Two equal and opposite charges seperated by a distance constitute an electric dipole. (e.g. NaCl molecule, Na^{+} Cl)

$2a$ - length of the dipole

(Net charge of a dipole is zero)

Dipole moment ($\vec{p}$) : The magnitude of dipole moment is equal to the product of the charge and the seperation between the charges. It is a vector quantity having a direction (by convension)from the negative to the positive charge. The Si unit of dipole moment is Cm

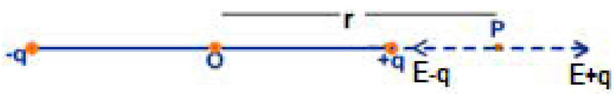


$|\vec{p}| = (2a)\,q$ or

$\vec{p} = 2a\,q\,\hat{p}$ ----------(9) ; where $\hat{p}$ is a unit vector from the –q to the +q charge.

Electric field at a point due to an electric dipole:

(i) at a point on its axial line



Consider an electric dipole of dipole moment p. P is a point at a ditance 'r' from the center of the dipole.

The electric field at the point P is the vector sum of electric fields due to the +q and –q charges.

i.e. $\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$ ----------(10)

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Since the directions of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ are opposite, we get

$$E = E_{+q} - E_{-q} \quad \text{-----------} \ (11)$$

$$E_{+q} = K \frac{q}{(r-a)^2} \quad \text{and} \quad E_{-q} = K \frac{q}{(r+a)^2}$$

$$E = Kq \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$E = Kq \left[\frac{4\,a\,r}{(r-a)^2(r+a)^2}\right]$$

$$E = K \left[\frac{2\ 2aq\ r}{(r^2-a^2)^2}\right] \quad \{ (r-a)^2(r+a)^2 = (r-a)(r-a)(r+a)(r+a) = (r^2-a^2)^2 \}$$

$$E = K \left[\frac{2\ p\ r}{(r^2-a^2)^2}\right] \quad \text{------------------}(12)$$

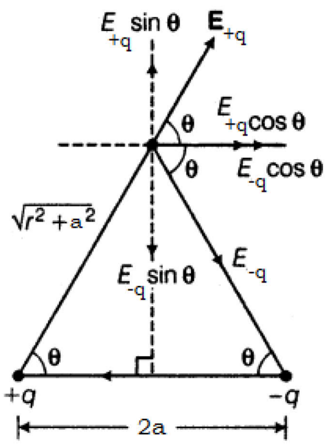
For a point dipole (a dipole whose size is negligible compared to other distances), $a << r$. i.e. a^2 is much smaller than r^2 and can be neglected.

$$E = K \frac{2\,p}{r^3} \quad \text{--------------}(13)$$

In vector form, $$\vec{E} = K \frac{2\,p}{r^3}\,\hat{p} \quad \text{------------}(14)$$

(ii) at a point on the equatorial plane :

Consider an electric dipole of dipole moment p. P is a point on its equatorial plane at a distance 'r' from the center of the dipole.



The electric field at the point P is the vector sum of electric fields due to the +q and –q charges.

i.e. $\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$

The the electric fields $\vec{E}_{+q}$ and $\vec{E}_{-q}$ can be resolved into two components as shown in the figure. The vertical (sine) components cancel each other where as the horizontal components add up.

$$\therefore E = \left|\vec{E}\right| = \left|\vec{E}_{+q}\right| \cos\theta + \left|\vec{E}_{-q}\right| \cos\theta$$

or, $$E = \left[E_{+q} + E_{-q}\right] \cos\theta \quad \text{-----} \ (15)$$

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From the figure,

$$E_{+q} = E_{-q} = K \frac{q}{r^2+x^2} \quad \text{and} \quad \cos\theta = \frac{a}{\sqrt{r^2+x^2}}$$

∴ (15)becomes,

$$E = K \frac{2q}{r^2+a^2} \frac{a}{\sqrt{r^2+a^2}}$$

$$\therefore \quad E = K \frac{2aq}{(r^2+a^2)^{\frac{3}{2}}} \text{ ----- (16)}$$

For a point dipole (a dipole whose size is negligible compared to other distances), $a << r$

i.e. a^2 is much smaller than r^2 and can be neglected.

∴ (16) becomes,

$$E = K \frac{2aq}{r^3} \text{ --------- (17)}$$

In vector form, $\vec{E} = -K \frac{2p}{r^3} \hat{p}$ --------(18)

The negative sign shows that the direction of electric field at an equatorial point is opposite to that of the dipole moment. For a point dipole,

$$\vec{E}_{\text{axis}} = K \frac{2p}{r^3} \hat{p} \quad \text{and}$$

$$\vec{E}_{\text{eq. Plane}} = -K \frac{p}{r^3} \hat{p}$$

$$\vec{E}_{\text{axis}} = -2(\vec{E}_{\text{eq.Plane}})$$

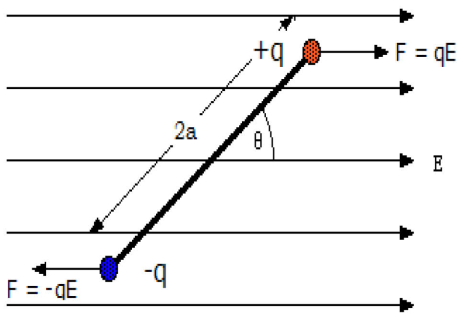
For same r,

$$\left|\left|\frac{\vec{E}_{\text{axis}}}{\vec{E}_{\text{eq.Plane}}}\right|\right| = 2:1$$

Q. What is the angle between electric field electric field on the axis and equatorial plane ? (Ans : 180°)

Electric Dipole in aUniform Electric Field

Consider an electric dipole of dipole moment p in a uniform electric filed of electric field strength E. The forces acting on both the +q and –q charges are equal(in magnitude) and opposite (in direction), the net force acting on the dipole is zero.



$$\vec{F}_{+q} = q\vec{E} \quad \text{and} \quad \vec{F}_{-q} = -q\vec{E}$$

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$$\vec{F}_{net} = 0$$

But these forces are not having the same line of action, the dipole experiences a torque which rotates the dipole.

Torque (τ) is given by,

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{------ (19)}$$

$$\vec{\tau} = 2a\,\hat{p} \; X \; q\vec{E} \qquad (\vec{\tau} = \vec{\tau}_{+q} + \vec{\tau}_{-q})$$

$$\vec{\tau} = 2aq\hat{p} \; X \; \vec{E}$$

$$\boxed{\vec{\tau} = \vec{p} \; X \; \vec{E}} \quad \text{------- (20)}$$

The magnitude of torque is given by,

$$\boxed{|\vec{\tau}| = p\,E\,sin\,\theta}$$ ---(21) and the direction is given by the Right-Hand Thumb rule.

Special Cases

(i) $\theta = 0^O$ (Stable Equilibrium)

From (21), the torque acting on the dipole is zero. ($sin\,0 = 0$). Since the net force and torque is zero, the dipole will be in both translational and rotational equilibrium. The potential energy of the dipole is minimum at $\theta = 0$. So it is called as the stable equilibrium position.

(ii) $\theta = 180\,^o$ (Unstable Equilibrium)

From (21), the torque acting on the dipole is zero. ($sin\,180 = 0$). Since the net force and torque is zero, the dipole will be in both translational and rotational equilibrium. The potential energy of the dipole is maximum at $\theta = 180$, hence called as the unstable equilibrium position.

(iii) $\theta = 90\,^o$

From (21), the torque acting on the dipole $\tau = p\,E$. ($sin\,90 = 1$). This is the maximum torque acting on the dipole.. The potential energy of the dipole is zero at $\theta = 90$.

Hence dipole moment is the torque acting on the dipole in a unit electric field when placed perpendicular to the field.

Electric Dipole in a Non - Uniform Electric Field

In a non-uniform electric field, the dipole will experience a net force.

Dipole moment is the torque or the moment of force acting on the dipole in a unit electric field when it is placed perpendicular to the field.

$$|\vec{\tau}| = \tau = p\,E\,sin\,\theta$$

$$E = 1, \; sin\,\theta = 1$$

$$\tau = p$$

Q. What should be the value of θ so that the torque on the dipole is half the maximum value? $(\theta = 30^0)$

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Continuous Charge Distribution

When large amount of charge is distributed over a region, discreet nature of charge can be ignored and it can be treated as a continuious charge distribution.

a) ***Linear Charge Density(λ)*** – Let Q amount of charge is distributed uniformly on a wire of length 'l'. Then the linear charge density is given by ,

$$\lambda = \frac{Q}{l} \quad ; \textbf{unit : C/m}$$

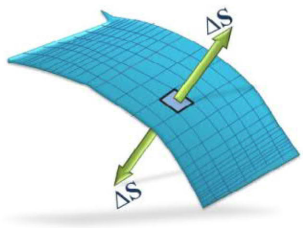
b) ***Surface Charge Density (σ)*** **-** Let Q amount of charge is distributed uniformly on a surface of area 'A'. Then the linear charge density is given by ,

$$\sigma = \frac{Q}{A} \quad ; \textbf{unit : C/m}^2$$

c) ***Volume Charge Density (ρ)*** **-** Let Q amount of charge is distributed uniformly on a body of volume 'V'. Then the volume charge density is given by ,

$$\rho = \frac{Q}{V} \quad ; \textbf{unit : C/m}^3$$

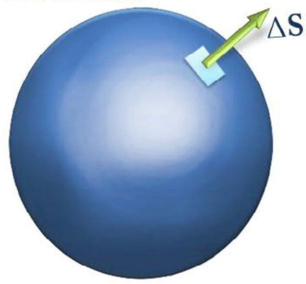
Area Vector : To represent the orientation of a surface, area is considered as a vector.(*Orientation of the plane in space can only be described by considering area as a vector & not a scalar).* Area vector has a magnitude equal to the area of the surface and a direction normal to the surface. By convention, for a closed surface, the direction of area vector is taken perpendicular and out of the surface.



Open Surface

$$\overrightarrow{\Delta S} = \Delta S\,\hat{n}$$; where ΔS is the area of the surface.

Closed Surface

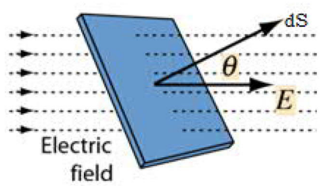


Electric Flux (ϕ)

Electric flux represents the measure of (number of) electric field lines passing through a given surface. The electric flux through an area ΔS is defined as

$$\Phi = \vec{E}\,.\,\overrightarrow{\Delta S} = E\,\Delta S\cos\theta \quad \text{--- (22)}$$

It is a scalar quantity and the SI unit is N m^2 C^{-1}.



When the electric filed lines are emerging out from a closed surface the electric flux is positive since we have $0 < \theta < 90$. When then electric filed lines are entering a closed surface the electric flux is negative since we have $90 < \theta < 180$.

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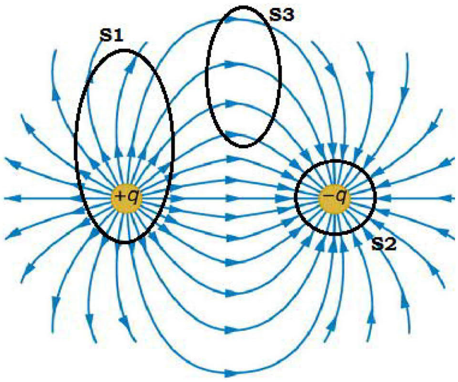
Q : Give the sign of electric flux for the closed surfaces S_1, S_2 and S_3 from the figure given below. (Ans : Positive, Negative, Zero)



Q : The flux through a shell of radius R enclosing a charge 'q' is Φ. What would be the flux through the shell if its radius increases to 2R ? (Ans : Φ)

Gauss's Law

It states that the surface integral of electric field over a closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

OR

The net electric flux passing through a closed surface is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

Mathematically,

$$\oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0} \quad \text{----(23)} \quad \textbf{OR}$$

$$\Phi = \frac{q}{\varepsilon_0}$$

--- (24) ; where ' q' is ther net charge enclosed by the surface.

Points regarding Gauss's law

1. Flux passing through a closed surface is independent of size and shape of the surface and depends only on the net charge enclosed.
2. Flux passing through a closed surface is independent of the position of the charge enclosed by the surface.
3. A Gaussian surface should never be drawn passing through a point charge as the electric filed at that point becomes infinite. But it can be drawn passing through a continuious charge distribution.
4. Gauss's law is applicable to only those fields which follows the inverse square law. (e.g. electrostatic, gravitational etc)

Applications of Gauss's law

(a) Field at a point due to a uniformly charged infinitely long straight wire.

(infinite means that the length of the wire is much larger than the distance between the wire and the point i,e, L >> r)

Consider a long, staight,uniformly charged wire of length '*L*' and charge '*Q*' having a linear charge density λ ($\lambda = \frac{Q}{L}$). P is a point at a distance 'r' from the wire.We consider a cylindrical* gaussian surface co-axial with the wire, having a lenghth 'l' and radius 'r'.The guassian surface is a combination of three surfaces S_1, S_2 and S_3. The total flux ϕ through the guassian surface is given by,

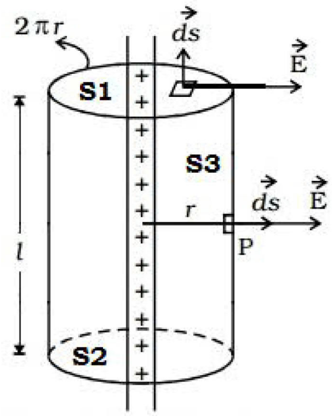


Fig 1.17 Infinitely long straight charged wire

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3$$

But $\Phi_1 = \Phi_2 = 0$ (since the area vector and electric field are perpendicular to each other, i.e. $\Phi = \vec{E}.\vec{\Delta S} = \Delta S \cos 90 = 0$)

$\therefore$ the net flux through the gaussian surface $\Phi = \Phi_3$.

But, $\Phi = \oint \vec{E}.\vec{dS}$

$= \oint E\ dS$ (on the curved surface S_3 E and dS are parallel)

$= E \oint dS$ (the electric field E is constant throughout the surface S_3)

$\Phi = E\ 2\pi r l$ --- (25)

From Gauss's law, we have,

$\Phi = \frac{q}{\varepsilon_0}$ ---- (26)

From (25) and (26), we get ;

$E\ 2\pi r l = \frac{q}{\varepsilon_0}$ (where q is the net charge encloased by the gaussian surface i.e. $q = \lambda l$)

$$E\ 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

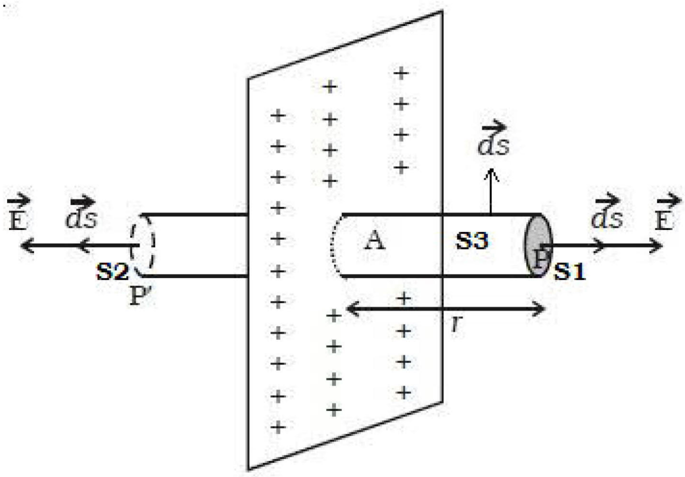
$$E = \frac{\lambda}{2\pi r\ \varepsilon_0}$$

In vector form,

$$\boxed{\vec{E} = \frac{\lambda}{2\pi r\ \varepsilon_0}\ \hat{n}}\ \text{------ (27)}$$

a) Electric field at a point due to a uniformly charged infinite# plane sheet.

Consider a plane, infinite, uniformly charged sheet having surface charge density 'σ' ($\sigma = \frac{Q}{A}$). P is a point at a distance 'r' perpendicular to the sheet. By symmetry, the electric field at all points having the same distance from the sheet will have the same magnitude and should be directed normal to the sheet.



We consider a cylindrical gaussian surface, penetrating the sheet, with its axis normal to the sheet. Let A is the base area of the gaussian surface.The net electric flux through the gaussain surface,

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3$$

But $\Phi_3 = 0$, since the electric field and area vector are perpendicular to each other.

∴ net electric flux through the gaussain surface,

$$\Phi = \Phi_1 + \Phi_2$$

or, $$\Phi = \oint \vec{E}.\vec{dS} + \oint \vec{E}.\vec{dS}$$

$$\Phi = EA + EA = 2\,E\,A \text{ ------(28)}$$

From Gauss;s law, we have,

$$\Phi = \frac{q}{\varepsilon_0} \text{ ------ (29)}$$

From (28) and (29), we have,

$$2\,E\,A = \frac{q}{\varepsilon_0}$$; where q is the net charge encloased by the guassian surface, equal to $q = \sigma A$

$$2\,E\,A = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

In vector form, $$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$; where 'n' is a unit vector perpendicular to the surface.

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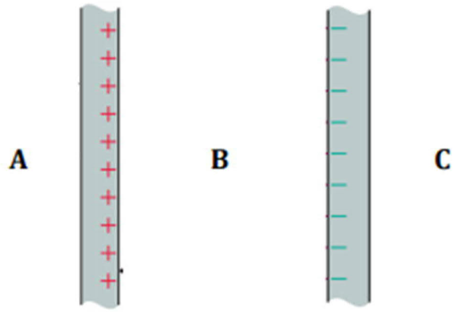
Q. Two infinite (very large) charged sheets are placed parallel. If the charge density of the plates are $+\sigma$ and $-\sigma$, find the magnitude and direction of electric field in regions A, B and C.

(Ans : 0, $\frac{\sigma}{\varepsilon_0}$, 0)

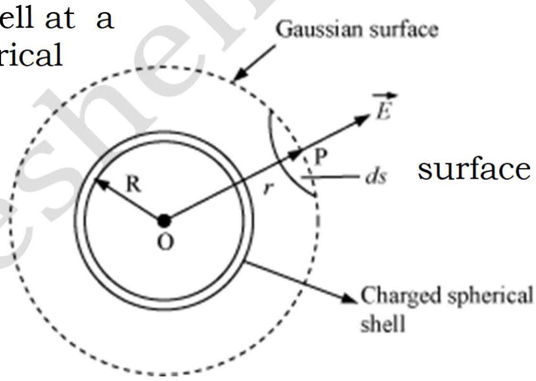


b) Electric field due to a uniformly charged spherical shell.

Consider a charged shell of radius 'R', uniformly charged to a value 'Q', having a uniform surface charge density 'σ'. By symmetery, the magnitude of electric field at all points at same distance should have the same value. In other words, the charged shell is having a spherical symmetry.

i. For a point outside the shell. ($r > R$)

Consider a point 'P' outside the charged shell at a distance 'r' from its centre. Assume a spherical gaussian surface having a radius 'r', concentric with the shell.



The net flux through the guasssian surface is given by,

$$\Phi = \oint \vec{E}.\vec{dS}$$

$$= \oint E.dS$$

$$= E \oint dS$$

$$= E \; 4\pi r^2 \quad \text{--(30)}$$

From Gauss;s law, we have,

$$\Phi = \frac{q}{\varepsilon_0} \quad \text{------(31)}$$

From (30) and (31), we get

$$E \; 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}} \quad \text{-------(32)}$$

But $q = \sigma A = \sigma \; 4\pi R^2$ ----(33)

Which is same as the relation for the electric filed due to a single charge. This means that a charged shell behaves as if its entire charge is concentrated at its centre.

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Substituting (33) in (32) we get,

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2} \quad \text{-----(34)}$$

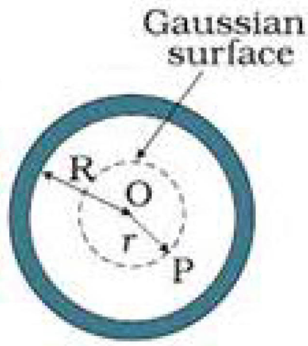
ii. For a point just outside surface of the shell ($r = R$)

For a point just outside the surface, $r = R$

(34) becomes,

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{-----(35)}$$

iii. For a point inside the shell ($r < R$)



Consider a point inside the charged shell at a distance $'r'$ from the centre.

Imagine a spherical gaussian surface of radius $'r'$.

From Gauss's law, we have,

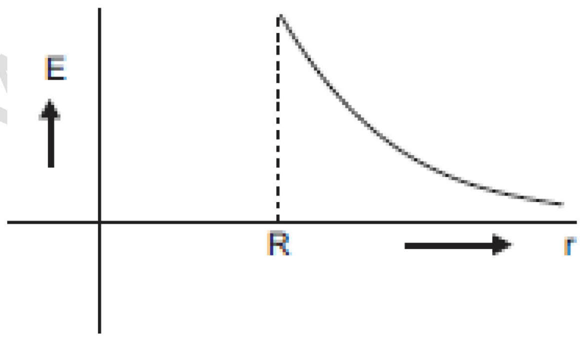
$$\Phi = \oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

But the charge enclosed by the Gaussian surface (q) is zero.

$$E = 0$$

The electric field inside a charged shell is zero.

Graphical representation of electric field by a charged shell



For a charged conducting sphere the electric filed is same as that of a shell. This is because, since the charges on a conductor resides on its surface, the conducting sphere behaves just like a charged shell.

In the case of a uniformly charges sphere (insulator) the electric field (inside & outside) can be found by using the Gauss's law.

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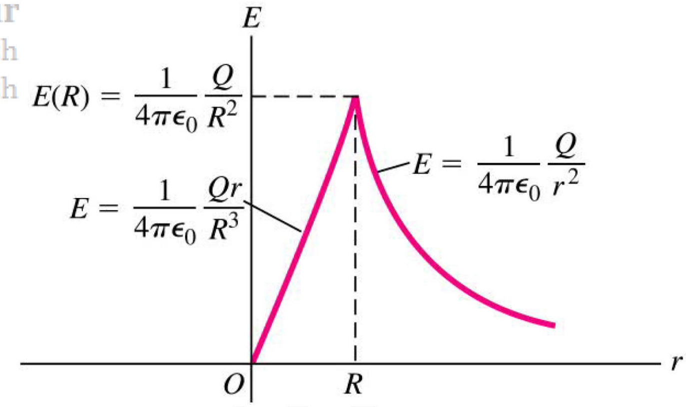
For a point 'p'	Charged Shell / Charged Sphere (Conductor)	Charged Sphere (Insulator)
Outside	$\frac{\sigma R^2}{\varepsilon_0 r^2}$	$\frac{\rho R^3}{3\varepsilon_0 r^2}$
On the surface	$\frac{\sigma}{\varepsilon_0}$	$\frac{\rho R}{3\,\varepsilon_0}$
Inside	Zero	$\frac{\rho r}{3\,\varepsilon_0}$

Graphical representation of electric field by a charged Insulating Sphere



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