12 Electrostatics

Section-A : JEE Advanced/ IIT-JEE

A 1. $\frac{\varepsilon_0 A}{d}\times V$; $\frac{2\varepsilon_0 A}{d}\times V$ 2. B 3. $180°, \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{4L^2}$ 4. $\frac{3V}{k+2}$

5. $-qEa$ 6. -8 7. $\frac{1}{4\pi\varepsilon_0}\frac{q\times q}{L^2}$

B 1. F 2. T 3. T 4. T 5. F 6. F

C

1. (b)	2. (b)	3. (b)	4. (d)	5. (d)	6. (b)	7. (b)
8. (c)	9. (a)	10. (b)	11. (b)	12. (c)	13. (c)	14. (c)
15. (d)	16. (c)	17. (a)	18. (d)	19. (b)	20. (c)	21. (c)
22. (a)	23. (b)	24. (a)	25. (a)	26. (d)	27. (c)	28. (d)
29. (c)	30. (a)	31. (c)	32. (d)	33. (c)	34. (c)	

D

1. (d)	2. (a, d)	3. (b)	4. (b, d)	5. (a)	6. (a)	7. (a, c, d)
8. (b)	9. (c)	10. (d)	11. (b, c)	12. (d)	13. (a, c)	14. (a, c)
15. (c, d)	16. (a,b,d)	17. (a)	18. (a, d)	19. (a, b, c, d)	20. (c, d)	21. (a, c, d)
22. (a, b, c)	23. (d)	24. (b, d)	25. (c, d)	26. (c)	27. (a, d)	28. (c)
29. (d)	30. (d)					

E 1. **(i)** Move towards centre; **(ii)** $Q=\frac{4\sqrt{3}q}{9}$, $3(2+\sqrt{3})K\frac{q^2}{a^2}$

2. **(i)** 60°; **(ii)** $mg=\pm k\frac{q_1q_2}{l^2}$; **(iii)** $N_1=\sqrt{3}\,mg$; $N_2=mg$ 3. $\frac{KQ(R+r)}{R^2+r^2}$ 4. 0.628 sec. 5. $\frac{3}{5}$

6. 8.48m 7. 3.16×10^{-9}C 8. $\frac{\pi}{2}\sqrt{\frac{ML}{2qE}}$

9. **(i)** $\frac{\sigma}{\epsilon_0}(a-b+c)$, $\frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b}-b+c\right)$, $\frac{\sigma}{\epsilon_0}\left(\frac{a^2-b^2+c^2}{c}\right)$ **(ii)** $c=a+b$

10. **(a)** $4a, (5a, 0)$ **(b)** $KQ\left[\frac{1}{3a-x}-\frac{2}{3a+x}\right]$ **(c)** $\sqrt{\frac{1}{4\pi\epsilon_0}\left(\frac{Qq}{2ma}\right)}$

11 **(a)** $\frac{3Q^2}{20\pi\epsilon_0 R}$ **(b)** $\frac{3GM^2}{5R}$, $1.5\times10^{32}J$ **(c)** $\frac{Q^2}{8\pi\epsilon_0 R}$

12. **(i)** 2×10^{-9} F, 1.21×10^{-5} J **(ii)** 4.84×10^{-5} J **(iii)** 1.1×10^{-5} J 13. $\sqrt{\frac{\lambda q}{2\epsilon_0 m}}$

14. 4.425×10^{-9}A. 15. $\frac{K_1K_2A\epsilon_0}{d(K_1-K_2)}\log\frac{K_1}{K_2}$

16. **(i)** 90×10^{-6}C, 210×10^{-6}C, 150×10^{-6}C **(ii)** 4.74×10^{-2} J, 1.8×10^{-2} J

17. **(a)** $\frac{1}{2}\times\frac{1}{4\pi\in_0 R}\times\left\{\frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]\right\}^2$ **(b)** $\frac{Q^2R}{2(4\pi\varepsilon_0)r^2}$ **18.** **(a)** $\frac{4a}{3}$ **(b)** $\frac{a}{\sqrt{3}}$

19. 3 m/s, 3×10^{-4} J **20.** $\frac{1}{4\pi\in_0}\frac{q^2}{a}.\frac{4}{\sqrt{6}}(3\sqrt{3}-3\sqrt{6}-\sqrt{2})$

21. **(a)** $\frac{1}{4\pi\in_0}\frac{pQ}{d^2}$ **(b)** $\frac{1}{4\pi\in_0}\frac{2pQ}{d^3}\hat{i}$ **22.** $\frac{q_0(\sigma_1-\sigma_2)a}{\sqrt{2}\in_0}$ **23.** $V\left(\frac{a}{3t}\right)^{1/3}$

F **1.** (A)-(p, r, s): (B)-(r, s); (C)-(p, q, t); (D)-(r, s) **2.** (a)
G **1.** (a) **2.** (b) **3.** (c) **4.** (c) **5.** (d)
H **1.** (a)
I **1.** 2 **2.** 3 **3.** 6 **4.** 6

Section-B : JEE Main/ AIEEE

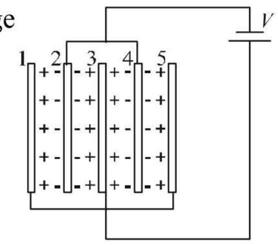
1. (a)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (d)
10. (b)	**11.** (d)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (b)
19. (c)	**20.** (c)	**21.** (c)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (a)
28. (b)	**29.** (c)	**30.** (a)	**31.** (c)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (c)	**36.** (a)
37. (d)	**38.** (c)	**39.** (c)	**40.** (c)	**41.** (b)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (a)
46. (d)	**47.** (a,b)	**48.** (c)	**49.** (a)	**50.** (c)				

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. On the plate 1 there is +ve charge

$$q=CV=\frac{\varepsilon_0 A}{d}\times V$$

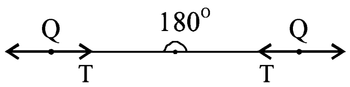


On the plate 4 the charge is

$$-2q=\frac{-2\varepsilon_0 A}{d}\times V$$

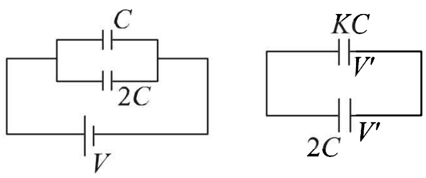
2. It is greatest at point *B* because at *B* the equipotential surfaces are closest.

3. There is no gravitational force acting. Only electrostatic force of repulsion is acting which will take the two balls as far as possible. The angle between the two strings will be 180°. The tension in the string will be equal to the electrostatic force of repulsion



$$T=\frac{1}{4\pi\varepsilon_0}\times\frac{Q\times Q}{(2L)^2}=\frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{4L^2}$$

4. Initially charge on capacitance $C=q_1=CV$
Charge on capacitance $C=q_2=2CV$



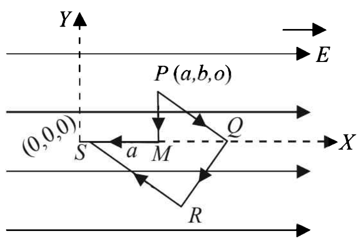
Finally charge on capacitance $C=q_1'=KCV'$
Charge on capacitance $2C=q_2'=2CV'$
Total charge will remain conserved

$$\therefore\quad CV+2CV=KCV'+2CV' \text{ or, } V'=\frac{3V}{K+2}$$

5. **NOTE :** Since electric field is conservative in nature, the work done by the field along *PQRS* will be same as along *PMS*

Work done from *P* to *M* $=\vec{F}.\overrightarrow{PM}$

$=F(PM)\cos 90°=0$



Work done from M to $S = \vec{F}.\vec{MS}$

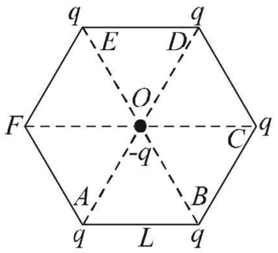
$= F(MS)\cos 180°$ $[\because F = qE]$
$= -qEa$

6. Electric potential $V = 4x^2$ volts
The electric potential changes only along x-axis,
We know that

$$E_x = \frac{-dV}{dx} \Rightarrow E_x = -\frac{d(4x^2)}{dx} = -8x$$

The electric field at point (1, 0, 2) will be (here $x = 1$)
$E_x = -8$ volt/m.

7.



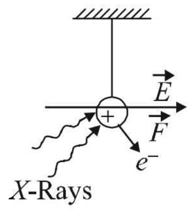
Force on (–q) due to charge at D will get cancelled out by force on (–q) due to change on A. Similarly force on –q due to charge at E will get cancelled out due to charge on B. So the net force will be because of charge on C $F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}$ directed from O to C.

B. True/ False

1. Electrostatic force is conservative in nature, therefore work done is path independent.

2. The metallic sphere which gets negatively charged gains electrons and hence its mass increases.
The metallic sphere which gets positively charged loses electrons and hence its mass decreases.

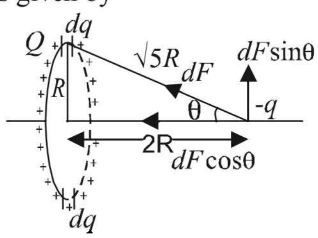
3. When high energy X-ray beam falls, it will knock out electrons from the small metal ball making it positively charged. Therefore the ball will be deflected in the direction of electric field.



4. The electric field produced between the parallel plate capacitor is uniform. The force acting on charged particle placed in an electric field is given by $F = qE$.
In the case of two protons, q and E are equal and hence force will be equal.

5. **KEY CONCEPT :** Force on charge $(-q)$ due to small charge dq situated at length $d\ell$ is given by

$$dF = k\frac{qdq}{5R^2}$$



Resolving this force into two parts $dF\cos\theta$ and $dF\sin\theta$ as shown in figure.

If we take another diametrically opposite length $d\ell$, the charge on it being dq. Then the force on charge $(-q)$ by this small charge dq will be

$$dF = k\frac{qdq}{5R^2}$$

Again resolving this force, we find $dF\sin\theta$ components of the two forces cancel out and $dF\cos\theta$ component adds up.

$\therefore$ The total force

$$F = \int_0^{2\pi R} dF\cos\theta = \int_0^{2\pi R}\frac{kqdq}{5R^2}\times\frac{2R}{\sqrt{5}R}$$

Charge on length $2\pi R = Q$

$\therefore$ Charge on length $d\ell = \frac{Qd\ell}{2\pi R} = dq$

$$\therefore \quad F = \int_0^{2\pi R}\frac{2kq}{5\sqrt{5}R^2}\times\frac{Qd\ell}{2\pi R}$$

$$= \frac{2kQq}{5\sqrt{5}\times 2\pi R^3}\times 2\pi R = \frac{2kQq}{5\sqrt{5}R^2}$$

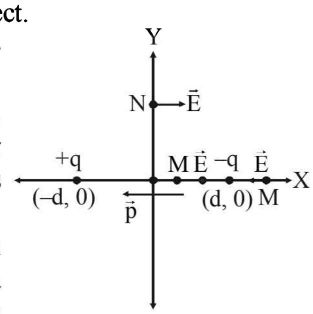
This is not an equation of simple harmonic motion.

6. For a particle to move in circular motion, we need a centripetal force which is not available.
The statement is false.

C. MCQs with ONE Correct Answer

1. **(b)** The potential at the surface of a hollow or conducting sphere is same as the potential at the centre of the sphere and any point inside the sphere.

2. **(b)** The two charges form an electric dipole. If we take a point M on the X-axis as shown in the figure, then the net electric field is in $-X$-direction.
$\therefore$ Option (a) is incorrect.
If we take a point N on Y-axis, we find net electric field along $+X$ direction. The same will be true for any point on Y-axis. (b) is a correct option.



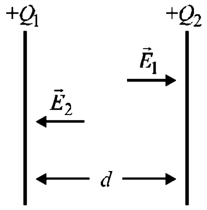
NOTE : For any point on the equatorial line of a dipole, the electric field is opposite to the direction of dipole moment.
(b) $W_{\infty - 0} = q(V_\infty - V_0) = q(0-0) = 0$
$\therefore$ (c) is incorrect. The direction of dipole moment is from –ve to + ve. Therefore (d) is incorrect.

3. **(b)** C and $2C$ are in parallel to each other.
$\therefore$ Resultant capacity $= (2C + C)$
$C_R = 3C$
Net potential $= 2V - V$
$V_R = V$

$\therefore$ Final energy $= \frac{1}{2}C_R(V_R)^2 = \frac{1}{2}(3C)(V)^2 = \frac{3}{2}CV^2$

4. **(d)** Within the capacitor,

$$E_1 = \frac{Q_1}{2\varepsilon_0 A}; E_2 = \frac{Q_2}{2\varepsilon_0 A};$$



where A = area of each plate
d = separation between two plate

$$E = E_1 - E_2 = \frac{1}{2\varepsilon_0 A}(Q_1 - Q_2)$$

Hence, $V = Ed$

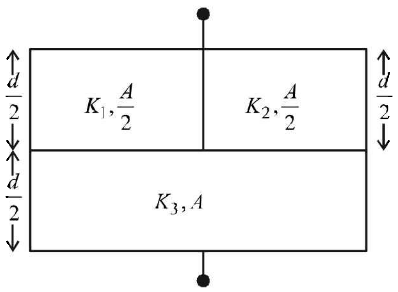
$$= \frac{1}{2}\frac{d}{\varepsilon_0 A}(Q_1 - Q_2) = \frac{Q_1 - Q_2}{2C}$$

5. **(d)** With the closing of switch S_3 and S_1 the negative charge on C_2 will attract the positive charge on C_2 thereby maintaining the negative charge on C_1. The negative charge on C_1 will attract the positive charge on C_1. No transfer of charge will take place. Therefore p.d across C_1 and C_2 will be 30 V and 20 V.

6. **(b)** Here we have $\frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$

$$\therefore \quad Q = -\frac{q\sqrt{2}}{\sqrt{2}+1} = -\frac{2q}{2+\sqrt{2}}$$

7. **(b)**



Let C_1 = Capacity of capacitor with K_1
C_2 = Capacity of capacitor with K_2
C_3 = Capacity of capacitor with K_3

$$\therefore \quad C_1 = K_1\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_1}{d}$$

$$\therefore \quad C_2 = K_2\left(\frac{A}{2}\right)\frac{\varepsilon_0 \times 2}{d} = \frac{A\varepsilon_0 K_2}{d}$$

$$\therefore \quad C_3 = K_3(A)\frac{\varepsilon_0 \times 2}{d} = \frac{2A\varepsilon_0 K_3}{d}$$

C_1 and C_2 are in parallel

$$\therefore \quad C_{eq} = \frac{A\varepsilon_0}{d}(K_1 + K_2)$$

C_{eq} and C_3 are in series

$$\therefore \quad \frac{1}{C} = \frac{d}{A\varepsilon_0(K_1+K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

But $C = \frac{KA\varepsilon_0}{d}$ for single equivalent capacitor

$$\therefore \quad \frac{d}{KA\varepsilon_0} = \frac{d}{A\varepsilon_0(K_1+K_2)} + \frac{d}{2A\varepsilon_0 K_3}$$

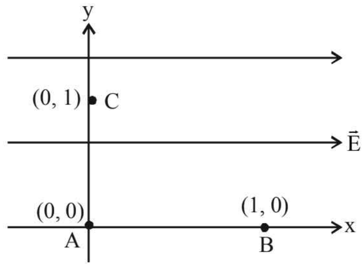
or $\frac{1}{K} = \frac{1}{K_1+K_2} + \frac{1}{2K_3}$.

8. **(c)** Electric field lines do not form closed loops. Therefore options (a), (b) and (d) are wrong.
Option (c) is correct. There is repulsion between similar charges.

9. **(a)** When S is closed, there will be no shifting of negative charge from plate A to B as the charge $-q$ is held by the charge $+q$. Neither there will be any shifting of charge from B to A.

10. **(b)** **NOTE :** As we move along the direction of electric field the potential decreases.

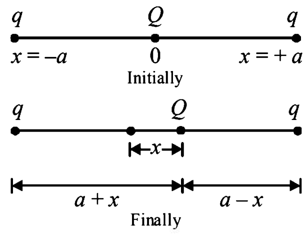
$\therefore \quad V_A > V_B$



11. **(b)** Initial energy

$$U_i = \frac{2Qq}{4\pi\varepsilon_0 a}$$

Final energy



$$U_f = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{a+x} + \frac{1}{a-x}\right] = \frac{2Qqa}{4\pi\varepsilon_0\left(a^2 - x^2\right)}$$

$$U_i - U_f = \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{a}{\left(a^2 - x^2\right)}\right]$$

$$= \frac{2Qq}{4\pi\varepsilon_0}\left[\frac{a^2 - x^2 - a^2}{a\left(a^2 - x^2\right)}\right] = \frac{-2Qqx^2}{4\pi\varepsilon_0 a^3}$$

when $x << a$ then x^2 is neglected in denominator.

$$U_i - U_f = \left(\frac{-Qq}{2\pi\varepsilon_0 a^3}\right)x^2$$

12. **(c)** Initially we know that

$$\Delta U = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$\Delta U = \frac{1}{2} \times \frac{C \times C}{2C}(V_1 - V_2)^2$$

$$\Delta U = \frac{C}{4}(V_1 - V_2)^2$$

13. (c) Electric field everywhere inside the metallic portion of shell is zero.

Hence options (a) and (d) are incorrect.

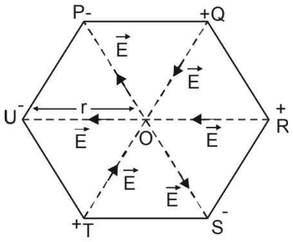
Electric field lines are always normal to a surface. Hence option (b) is incorrect. Only option (c) represents the correct answer.

14. (c) $|\vec{E}| = \frac{Kq}{r^2}$

Electric field due to P on O is cancelled by electric field due to S on O.

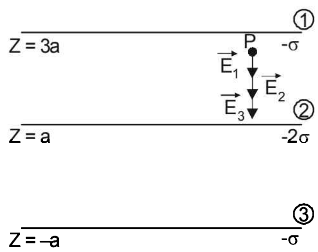
Similarly Electric field due to Q on O is cancelled by electric field due to T and O.

The electric field due to R on O is in the same direction as that of U on O. Therefore the net electric field is $2\vec{E}$.



15. (d) The flux through the Gaussian surface is due to the charges inside the Gaussian surface. But the electric field on the Gaussian surface will be due to the charges present in side the Gaussian surface and outside it. It will be due to all the charges.

16. (c) Figure shows the electric fields due to the sheets 1, 2 and 3 at point P. The direction of electric fields are according to the charge on the sheets (away from positively charge sheet and towards the negatively charged sheet and perpendicular).



The total electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

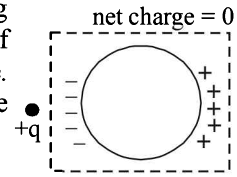
$$= E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k})$$

$$= \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k}) \quad = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

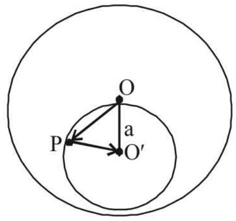
17. (a) When a charge density is given to the inner cylinder, the potential developed at its surface is different from that on the outer cylinder. This is because the potential decreases with distance for a charged conducting cylinder when the point of consideration is outside the cylinder.

But when a charge density is given to the outer cylinder, it will change its potential by the same amount as that of the inner cylinder. Therefore no potential difference will be produced between the cylinders in this case.

18. (d) When a positive point charge is placed outside a conducting sphere, a rearrangement of charge takes place on the surface. But the total charge on the sphere is zero as no charge has left or entered the sphere.



19. (b) Let us consider a uniformly charged solid sphere without any cavity. Let the charge per unit volume be σ and O be the centre of the sphere. Let us consider a uniformly charged sphere of negative charged density σ having its centre at O'. Also let OO' be equal to a.



Let us consider an arbitrary point P in the small sphere. The electric field due to charge on big sphere

$$\vec{E}_1 = \frac{\sigma}{3\varepsilon_0}\overrightarrow{OP}$$

Also the electric field due to small sphere

$$\vec{E}_2 = \frac{\sigma}{3\varepsilon_0}\overrightarrow{PO'}$$ ∴ The total electric field

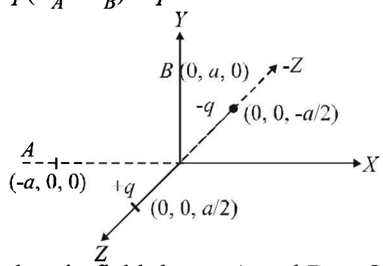
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{3\varepsilon_0}[\overrightarrow{OP} + \overrightarrow{PO'}] = \frac{\sigma}{3\varepsilon_0}\overrightarrow{OO'}$$

Thus electric field will have a finite value which will be uniform.

20. (c) The charges make an electric dipole. A and B points lie on the equatorial plane of the dipole.

Therefore, potential at A = potential at $B = 0$

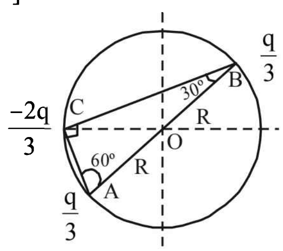
$W = q(V_A - V_B) = q \times 0 = 0$



21. (c) The electric field due to A and B at O are equal and opposite producing a resultant which is zero. The electric field at O due to C is

$$E = \frac{1}{4\pi\epsilon_0}\frac{2q/3}{R^2} = \frac{q}{6\pi\epsilon_0 R^2}.$$

∴ Option [A] is not correct. The electric potential at O is



$$V_O = K\left[\frac{+q/3}{R}\right] + K\left[\frac{+q/3}{R}\right] + K\left[\frac{-2q/3}{R}\right] = 0$$

∴ Option [D] is wrong

In $\Delta ABC \dfrac{AC}{AB} = \sin 30° \Rightarrow AC = \dfrac{AB}{2} = R$

Also $\dfrac{BC}{AB} = \sin 60° \Rightarrow \quad BC = \dfrac{\sqrt{3}AB}{2} = \sqrt{3}R$

Potential energy of the system

$$K\left[\frac{(q/3)(2/3)}{2R}\right] + K\left[\frac{(q/3)(-2q/3)}{R}\right] + K\left[\frac{(q/3)(-2q/3)}{\sqrt{3}R}\right]$$

$$= \frac{kq^2}{9R}\left[\frac{1}{2} - 2 - \frac{2}{\sqrt{3}}\right] \neq 0$$

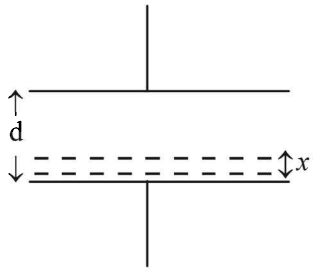
$\therefore$ Option [B] is wrong

Magnitude of force between B and C is

$$F = \frac{1}{4\pi \epsilon_0} \frac{(2q/3)(q/3)}{(\sqrt{3}R)^2} = \frac{q^2}{54\pi \epsilon_0 R^2}$$

22. (a) Let the level of liquid at an instant of time 't' be x. Then

$$v = -\frac{dx}{dt} \Rightarrow dx = -vdt$$



$$\Rightarrow \int_{d/3}^{x} dx = -v\int_0^t dt$$

$$\Rightarrow x - \frac{d}{3} = -vt$$

$$\Rightarrow x = \frac{d}{3} - vt$$

Also the capacitance can be considered as an equivalent of two capacitances in series such that

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{\frac{\epsilon_0 A}{d-x}} + \frac{1}{\frac{\epsilon_0 AK}{x}} = \frac{d-x}{\epsilon_0 A} + \frac{x}{\epsilon_0 AK}$$

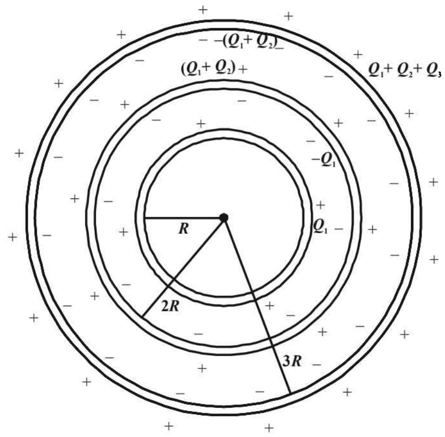
$$\therefore C_{eq} = \frac{\epsilon_0 AK}{Kd + x(1-K)}$$

But $A = 1$, $K = 2$ and $x = \dfrac{d}{3} - vt$

$$\therefore C_{eq} = \frac{\epsilon_0 \times 1 \times 2}{2d + \left[\frac{d}{3} - vt\right](1-2)} = \frac{6\epsilon_0}{5d + 3vt}$$

$$\therefore \text{Time constant } \tau = RC_{eq} = \frac{6R\epsilon_0}{5d + 3vt}$$

23. (b) The charges on the surfaces of the metallic spheres are shown in the diagram. It is given that the surface charge densities on the outer surfaces of the shells are equal. Therefore



$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2} = x(\text{say})$$

$$\therefore \quad Q_1 = 4\pi R^2 x$$

$$Q_1 + Q_2 = 4\pi(2R)^2 k = 4[4\pi R^2 x]$$

$$\Rightarrow Q_2 = 4[4\pi R^2 x] - Q_1$$

$$= 4[4\pi R^2 x] - 4\pi R^2 x = 3[4\pi R^2 x]$$

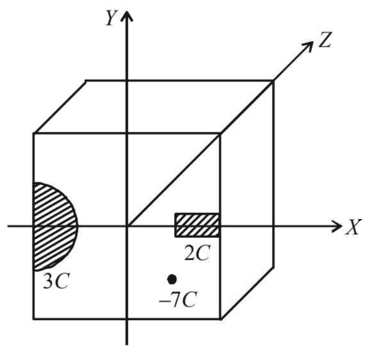
Also $Q_1 + Q_2 + Q_3 = 4\pi(3R)^2 x = 9[4\pi R^2 x]$

$$\therefore \quad Q_3 = 9[4\pi R^2 x] - Q_1 - Q_2 = 9[4\pi R^2 x] - [4\pi R^2 x] - 3[4\pi R^2 x] = 5[4\pi R^2 x]$$

$$\Rightarrow Q_1 : Q_2 : Q_3 = 1 : 3 : 5$$

24. (a) From the figure it is clear that the charge enclosed in the cubical surface is $3C + 2C - 7C = -2C$. Therefore the electric flux through the cube is

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{-2C}{\varepsilon_0}$$



25. (a) The electrostatic pressure at a point on the surface of a uniformly charged sphere $= \dfrac{\sigma^2}{2\epsilon_0}$

$\therefore$ The force on a hemispherical shell $= \dfrac{\sigma^2}{2\epsilon_0} \times \pi R^2$

26. (d) **When the electric field is on**

Force due to electric field = weight

$qE = mg$

$qE = \frac{4}{3}\pi R^3 \rho g \quad \therefore q = \frac{4\pi R^3 \rho g}{3E}$ ...(i)

When the electric field is switched off

Weight = viscous drag force

$mg = 6\pi\eta R v_t$

$\frac{4}{3}\pi R^3 \rho g = 6\pi\eta R v_t \quad \therefore R = \sqrt{\frac{9\eta v_t}{2\rho g}}$ ...(ii)

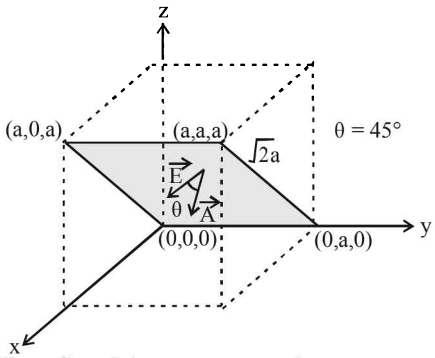
From (i) & (ii) $q = \frac{4}{3}\pi\left[\frac{9\eta v_t}{2\rho g}\right]^{\frac{3}{2}} \times \frac{\rho g}{E}$

$= \frac{4}{3} \times \pi\left[\frac{9 \times 1.8 \times 10^{-5} \times 2 \times 10^{-3}}{2 \times 900 \times 9.8}\right]^{\frac{3}{2}} \times \frac{900 \times 9.8 \times 7}{81\pi \times 10^5}$

$= 7.8 \times 10^{-19}$ C

27. **(c)** Given $\vec{E} = E_o \hat{x}$

This shows that the electric field acts along $+x$ direction and is a constant. The area vector makes an angle of 45° with the electric field. Therefore the electric flux through the shaded portion whose area is $a \times \sqrt{2}a = \sqrt{2}a^2$ is $\phi = \vec{E}.\vec{A} = EA\cos\theta = E_0(\sqrt{2}a^2)$ cos 45° $= E_0(\sqrt{2}a^2) \times \frac{1}{\sqrt{2}} = E_0 a^2$



28. **(d)** **When S and 1 are connected**

The 2μF capacitor gets charged. The potential difference across its plates will be V.

The potential energy stored in 2 μF capacitor

$U_i = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times V^2 = V^2$

When S and 2 are connected

The 8μF capacitor also gets charged. During this charging process current flows in the wire and some amount of energy is dissipated as heat. The energy loss is

$\Delta U = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$

Here, $C_1 = 2\mu F, C_2\, 8\,\mu F, V_1 = V, V_2 = 0$

$\therefore \Delta U = \frac{1}{2} \times \frac{2 \times 8}{2+8}(V-0)^2 = \frac{4}{5}V^2$

The percentage of the energy dissipated $= \frac{\Delta U}{Ui} \times 100$

$= \frac{\frac{4}{5}V^2}{V^2} \times 100 = 80\%$

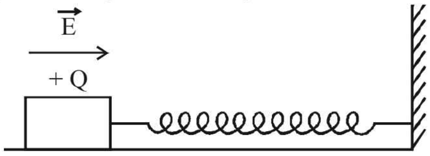
29. **(c)** The pattern of field lines shown in option (c) is correct because

(a) a current carrying toroid produces magnetic field lines of such pattern

(b) a changing magnetic field with respect to time in a region perpendicular to the paper produces induced electric field lines of such pattern.

30. **(a)** The frequency of SHM performed by wooden block is

$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

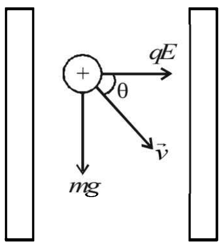


when electric field is switched on, the value of k and m is not affected and therefore the frequency of SHM remains the same. But as an external force QE starts acting on the block towards right, the mean position of SHM shifts towards right by $\frac{QE}{k}$

correct option is (a).

Note : In SHM if a constant additional force is applied then it only shift the quilibrium position and does not change the frequency fo SHM.

31. **(c)** The two forces acting on the proton just after the release are shown in the figure. In this situation



$qE = mg \qquad [\therefore \theta = 45°]$

$\therefore \quad q\left(\frac{V}{d}\right) = mg$

$\therefore \quad V = \frac{mgd}{q} = \frac{1.67 \times 10^{-27} \times 10 \times 10^{-2}}{1.6 \times 10^{-19}} = 10^{-9}V$

32. **(d)** For a thin uniformly positive charged spherical shell

(i) Inside the shell at any point

$E = O$ and $V = \frac{1}{4\pi \epsilon_0}\frac{q}{R}$ = constt.

where q = charge on sphere

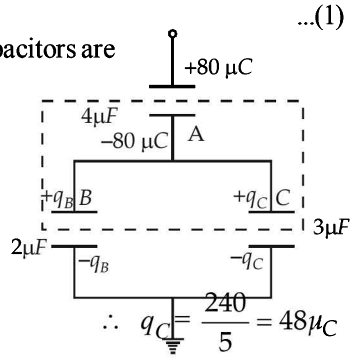
R = Radius of sphere

(ii) Outside the shell at any point at any distance r from the centre $E \propto \frac{1}{r^2}$ and $V \propto \frac{1}{r}$

33. **(c)** The total charge on plate A will be $-80\ \mu C$. If q_B and q_C be the charges on plate B and C then

$q_B + q_C = 80\ \mu C$ ...(1)

Also $2\mu F$ and $3\mu F$ capacitors are in parallel. Therefore,



$$\frac{q_B}{2} = \frac{q_C}{3}$$

$$\therefore \frac{80 - q_C}{2} = \frac{q_C}{3}$$

$$\therefore 240 - 3q_C = 2q_C \qquad \therefore q_C = \frac{240}{5} = 48\mu C$$

This charge will obviously be positive.

34. **(c)** $E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2}$;

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{R^2};\ E_3 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q/2}{R^2}$$

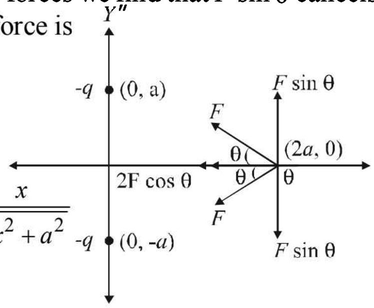
Clearly $E_2 > E_1 > E_3$

where $Q/2$ is the charge enclosed in a sphere of radius R concentric with the given sphere.

$$\left[\frac{4Q}{\frac{4}{3}\pi(2R)^3} = \frac{Q'}{\frac{4}{3}\pi R^3}\right]$$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(d)** Let us consider the positive charge Q at any instant of time t at a distance x from the origin. It is under the influence of two forces $\vec{F_1}$ $(= F)$ and $\vec{F_2}$ $(= F)$. On resolving these two forces we find that $F \sin\theta$ cancels out. The resultant force is



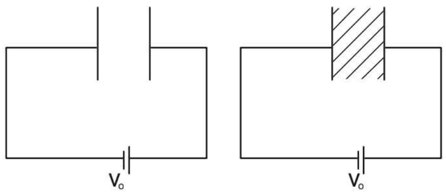
$$F_R = 2F\cos\theta$$

$$= 2 \times \frac{kQq}{(x^2 + a^2)} \times \frac{x}{\sqrt{x^2 + a^2}}$$

$$= \frac{2kQqx}{(x^2 + a^2)^{3/2}}$$

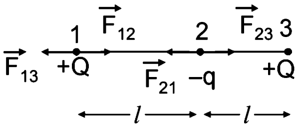
Since F_R is not proportional to x, the motion is NOT simple harmonic. The charge Q will accelerate till the origin and gain velocity. At the origin the net force is zero but due to momentum it will cross the origin and more towards left. As it comes on negative x-axis, the force is again towards the origin.

2. **(a, d)**



(i) P.d. $= V_0$ Capacitance $= C$	P.d. $= V_0$ Capacitance $= KC$ [K is the dielectric constant of slab $K > 1$]
(ii) $Q_0 = CV_0$	New charge $= KCV_0$
(iii) Potential Energy $= \frac{1}{2}CV_0^2$	New potential energy $= \frac{1}{2}KCV_0^2$
(iv) $E = \frac{V_0}{d}$	$E = \frac{V_0}{d}$

3. **(b)** q has to be negative for equilibrium.



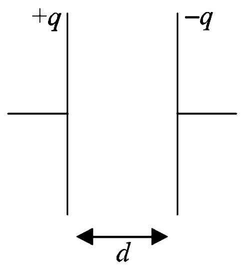
Considering equilibrium of 1

$$F_{13} = F_{12}$$

$$\frac{KQ \times Q}{(2\ell)^2} = \frac{KQ(-q)}{\ell^2} \quad \therefore q = -\frac{Q}{4}$$

4. **(b, d)**

Charge on plate is q

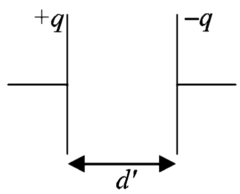


$$C = \frac{\varepsilon_0 A}{d}$$

$$q = CV \Rightarrow V = \frac{q}{C},$$

$$U = \frac{1}{2}q \times V$$

Charge on plate is q



$$C' = \frac{\varepsilon_0 A}{d'} \Rightarrow C' < C,$$

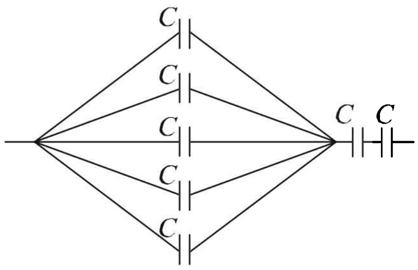
$$V' = \frac{q}{C'} \Rightarrow V' > V$$

$$U' = \frac{1}{2}qV' \Rightarrow U' = U$$

5. **(a)** The potential inside the shell will be the same everywhere as on its surface. As we add $-3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore the potential difference remains the same.

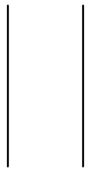
6. **(a)** The equivalent capacitance

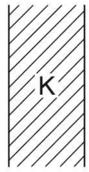
$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2 \times 5} = \frac{11}{10} \Rightarrow C_{eq} = \frac{10}{11}\mu F$$



7. (a, c, d)

As $C = \frac{\varepsilon_0}{d}A$ | $C' = \frac{K\varepsilon_0 A}{d}$





$V' = \frac{V}{K}$

$Q = CV$

$= \frac{\varepsilon_0 A}{d} \times V$ | $Q = \frac{\varepsilon_0 AV}{d} = C'V'$

Q will remain same as no charge is leaving or entering the plates during the process of slab insertion

Now, $Q = C'V' = C'E'd$

$$E' = \frac{Q}{C'd} = \frac{\frac{\varepsilon_0 AV}{d}}{\frac{K\varepsilon_0 A}{d}} \times \frac{1}{d} = \frac{V}{Kd}$$

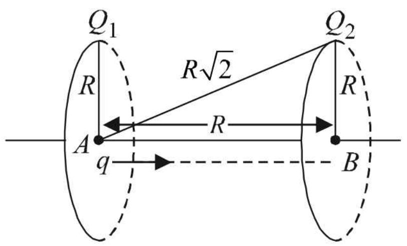
Work done is the change in energy stored

$$W = \frac{1}{2}CV^2 - \frac{1}{2}C'V'^2$$

$$= \frac{1}{2}\frac{\varepsilon_0 AV^2}{d} - \frac{1}{2}\frac{K\varepsilon_0 A}{d} \times \left(\frac{V}{K}\right)^2 \quad \left[\because V' = E'd = \frac{V}{K}\right]$$

$$W = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\left[1 - \frac{1}{K}\right]$$

8. (b) The work done in moving a charge from A to B



$W = (T.P.E.)_A - (T.P.E.)_B$ where $T.P.E.$ = Total Potential Energy

$$(T.P.E.)_A = \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right) \times q + \left(\frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$$

$$(T.P.E.)_B = \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right)q + \left(\frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

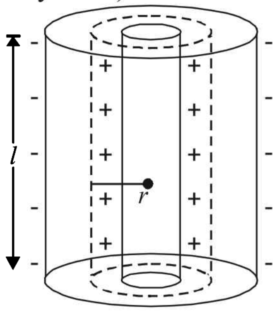
$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$$

$$\therefore \quad W = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$$

$$= \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)$$

9. (c) Let λ be the charge per unit length. Let us consider a Gaussian surface (dotted cylinder).

Applying Gauss's law



$$\phi = \oint \vec{E}.\vec{ds} = \frac{\lambda \ell}{\varepsilon_0}$$

For the flat portions of Gaussian surface, the angle between electric field and surface is 90°. Hence flux through flat portions is zero.

NOTE : By symmetry, the electric field on the curved surface is same throughout.

The angle between $\vec{E}$ and $\vec{ds}$ is 0° (for curved surface)

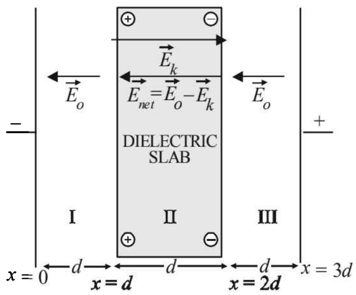
$$\Rightarrow \quad E\int ds = \frac{\lambda\ell}{\varepsilon_0} \quad \Rightarrow \quad E \times 2\pi r\ell = \frac{\lambda\ell}{\varepsilon_0}$$

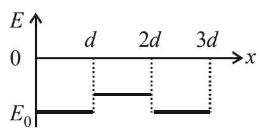
$$\Rightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \Rightarrow \quad E \propto \frac{1}{r}$$

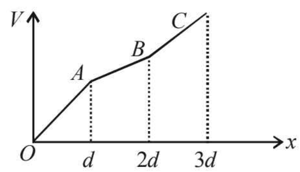
10. (d) The electric lines of force cannot enter the metallic sphere as electric field inside the solid metallic sphere is zero. Also, the origination and termination of the electric lines of force from the metallic surface is normally (directed towards the centre).

11. (b, c)

In region I and III, there will be electric field $\vec{E}_0$ directed from positive to negative. In region II, due to orientation of dipoles, there is an electric field $\vec{E}_k$ present in opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field is $\vec{E}_0 - \vec{E}_k$ in the direction of $\vec{E}_0$ as shown in the diagram. ($\because E_0 > E_k$)







NOTE : When one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the potential increase. Since in region II, the electric field is less as compared to I and III therefore the increase in potential will be less but there has to be increase in potential in all the regions from $x = 0$ to $x = 3d$. Also where E is uniform, $\frac{dV}{dx} = \text{constt.}$

12. (d) Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + ...\right]$$

$$V = \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

13. (a, c)
Let Q be the charge on the ring, the negative charge $-q$ is released from point $P(0, 0, Z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{QZ_0}{(R^2 + Z_0^2)^{3/2}}$$

Therefore, force on charge P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(R^2 + Z_0^2)^{3/2}}\cdot Z_0 \qquad ...(1)$$

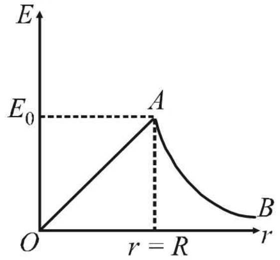
Similarly, when it crosses the origin, the force is again towards centre O.
Thus the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.
Secondly if $Z_0 << R$, $(R^2 + Z_0^2)^{3/2} \approx R^3$

$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}\cdot Z_0 \qquad \text{[From equation 1]}$$

i.e. the restoring force $F_e \propto -Z_0$. Hence the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position).

14. (a, c)
KEY CONCEPT : The expressions of the electric field inside the sphere $(r < R)$ $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{R^3}r$; outside the sphere

$(R < r < \infty)$ $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$



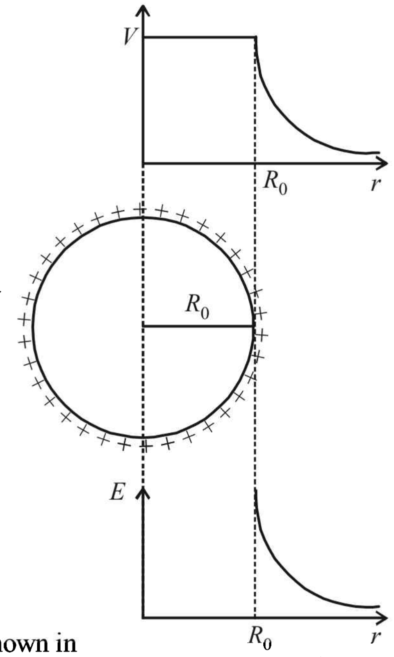
Hence, E increases for $r < R$ and decreases for $R < r < \infty$.

15. (c, d)
When two points are connected with a conducting path in electrostatic condition, then the potential of the two points is equal. Thus potential at A = Potential at B
(c) is the correct option.
Option (d) is a result of Gauss's law

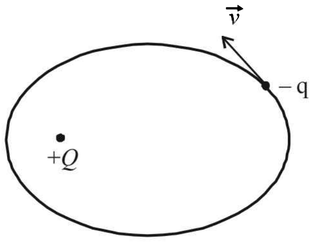
Total electric flux through cavity $= \frac{q}{\epsilon_0}$

Option (a) and (b) are dependent on the curvature which is different at points A and B.

16. (a,b,d)
(a) The whole charge Q will be enclosed in a sphere of diameter $2R_0$.
(b) Electric field E = zero inside the sphere. Hence electric field is discontinued at $r = R_0$.
(c) Changes in V and E are continuously present for $r > R_0$. Option (c) is incorrect.
(d) For $r < R_0$, the potential V is constant and the electric intensity is zero. Obviously, the electrostatic energy is zero for $r < R_0$.



17. (a) The situation is shown in the figure which is similar to a planet revolving around sun. The distance of $-q$ from $+Q$ is changing, therefore, force between the charges will change.



The speed of the charge $-q$ will be greater when the charge is nearer to $+Q$ as compared to when it is far. Therefore, the angular velocity of charge $-q$ is also variable. The direction of the velocity changes continously, therefore, linear momentum is also variable. The angular momentum of $(-q)$ about Q is constant because the torque about $+Q$ is zero.

18. (a,d)
The electric field lines are orginating from Q_1 and terminating on Q_2. Therefore Q_1 is positive and Q_2 is negative.
As the number of lines associated with Q_1 is greater than that associated with Q_2, therefore $|Q_1| > |Q_2|$.
Option (a) is correct.
At a finite distance on the left of Q_1, the electric field intensity cannot be zero because the electric field created by Q_1 will be greater than Q_2. This is because the magnitude of Q_1 is greater and the distance smaller $\left[E \propto \frac{Q}{r^2}\right]$

At a finite distance to the right of Q_2, the electric field is zero. Here, the electric field created by Q_2 at a particular point will cancel out the electric field created by Q_1.

19. (a, b, c, d)
Electric field inside a spherical metallic shell with charge on the surface is always zero. Therefore option [a] is correct.
When the shells are connected with a thin metal wire then electric potentials will be equal, say V.

$$\therefore \frac{1}{4\pi\epsilon_0}\frac{Q_A}{R_A} = \frac{1}{4\pi\epsilon_0}\frac{Q_B}{R_B} = V$$

As $R_A > R_B$ therefore $Q_A > A_B$. option [b] is also correct.

As $\dfrac{\sigma_A}{\sigma_B} = \dfrac{\dfrac{Q_A}{4\pi R_A^2}}{\dfrac{Q_B}{4\pi R_B^2}} = \dfrac{R_B^2}{R_A^2} \times \dfrac{Q_A}{Q_B} = \dfrac{R_B^2}{R_A^2} \times \dfrac{4\pi \epsilon_0 R_A V}{4\pi \epsilon_0 R_B V}$

$\therefore \dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ Option (c) is also correct

Also $E_A = \dfrac{\sigma_A}{\epsilon_0}$ & $E_B = \dfrac{\sigma_B}{\epsilon_0}$

$\dfrac{E_A}{E_B} = \dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A} < 1 \quad \therefore E_A < E_B$

Option (d) is also correct

20. **(c, d)**

(a) is not correct because it is valid only when $E \propto r^{-2}$

(b) is not correct

(c) is correct as between two point charges we will get a point where the electric field due to the two point charges cancel out each other.

(d) is correct when the work done is without accelerating the charge.

21. **(a, c, d)**

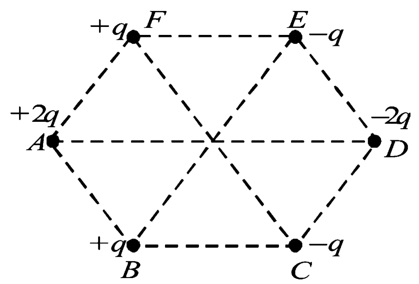
The electric flux passing through $x = +\dfrac{a}{2}$,

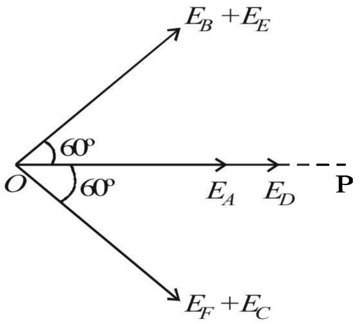
$x = -\dfrac{a}{2}, z = +\dfrac{a}{2}$ is same due to symmetry.

The net electric flux through the cubical region is

$\dfrac{-q+3q-q}{\varepsilon_0} = \dfrac{q}{\varepsilon_0}$.

22. **(a, b, c)**





Here $\dfrac{|\vec{E_A}|}{2} = |\vec{E_B}| = |\vec{E_C}| = \dfrac{|\vec{E_D}|}{2} = |\vec{E_E}| = |\vec{E_F}| = K$

$\therefore \; E_O = E_A + E_D + (E_F + E_C)\cos 60° + (E_B + E_C)\cos 60°$

$= 2K + 2K + (K+K) \times \dfrac{1}{2} + (K+K) \times \dfrac{1}{2} = 6K$

The electric potential at O is

$$V_O = \frac{1}{4\pi \varepsilon_0 L}[2q + q + q - q - q - 2q] = 0$$

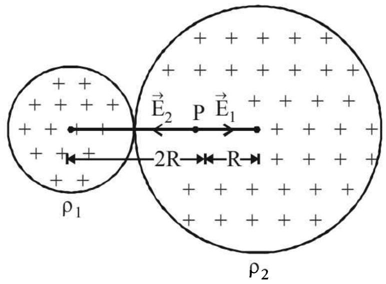
PR is perpendicular bisector (the equatorial line) for the electric dipoles AB, FE and BC. Therefore the electric potential will be zero at any point on PR.

At any point ST, the electric field will be directed from S to T. The potential decreases along the electric field line.

23. **(b, d)**

Electric field E_1 due to smaller sphere at P is

$$E_1 = \frac{1}{4\pi\varepsilon_0} \frac{\rho_1 \times \frac{4}{3}\pi R^3}{(2R)^2}$$



$$E_1 = \frac{1}{4\pi \epsilon_0} \times \frac{\rho_1 \pi R}{3} = \frac{\rho_1 R}{4 \epsilon_0 \times 3}$$

Electric field E_2 due to bigger sphere at P is

$$E_2 = \frac{\rho_2 R}{3 \epsilon_0}$$

As $E_1 = E_2 \quad \therefore \dfrac{\rho_1 R}{4 \epsilon_0 \times 3} = \dfrac{\rho_2 R}{3 \epsilon_0} \Rightarrow \dfrac{\rho_1}{\rho_2} = 4$

Option (d) is correct.

24. **(b, d)**

Step 1 : When S_1 is pressed. The capacitor C_1 gets charged such that its upper plate acquires a positive charge $+2CV_0$ and lower plate $-2CV_0$.

Step 2 : When S_2 is pressed (S_1 open). As $C_1 = C_2$ the charge gets distributed equal. The upper plates of C_1 and C_2 now take charge $+CV_0$ each and lower plate $-CV_0$ each.

(b) and (d) are correct option.

25. **(c, d)**

Let us consider a point P on the overlapping region. The electric field intensity at P due to positively charged sphere

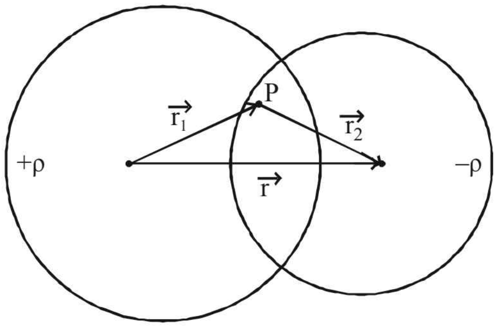
$= \dfrac{\rho \vec{r_1}}{3 \epsilon_0}$

The electric field intensity at P due to negatively charged sphere $= \dfrac{\rho \vec{r_2}}{3 \epsilon_0}$. The total electric field,

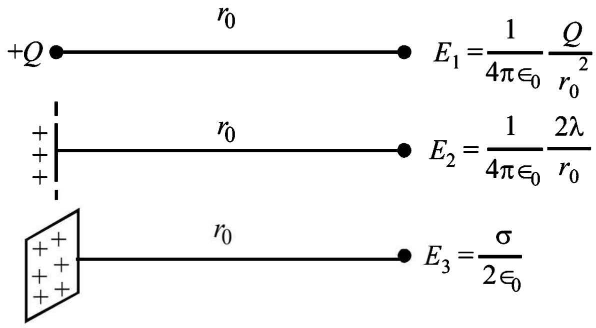
$$\vec{E} = \frac{\rho \vec{r_1}}{3 \epsilon_0} + \frac{\rho \vec{r_2}}{3 \epsilon_0} = \frac{\rho}{3 \epsilon_0}\left[\vec{r_1} + \vec{r_2}\right]$$

$$\vec{E} = \frac{\rho}{3 \epsilon_0}\vec{r}$$

Therefore the electric field is same in magnitude and direction option (c) and (d) are correct.



26. (c)



$E_1 = E_2$ (Given)

$$\frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0}$$

$\therefore Q = 2\lambda r_0$ ...(1)

$E_2 = E_3$ (Given)

$$\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0} = \frac{\sigma}{2\epsilon_0} \Rightarrow r_0 = \frac{\lambda}{\sigma\pi}$$

$\therefore$ (b) is incorrect

$E_1 = E_3$ (Given)

$$\therefore \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2} = \frac{\sigma}{2\epsilon_0} \Rightarrow Q = 2\pi\sigma r_0^2$$

$\therefore$ (a) is incorrect

$$\text{Now } E_1(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{4Q}{r_0^2}$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{4\times 2\lambda r_0}{r_0^2} = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$$

$$\text{and } 2E_2(r_0/2) = 2\left[\frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0}\right] = \frac{1}{4\pi\epsilon_0}\frac{8\lambda}{r_0}$$

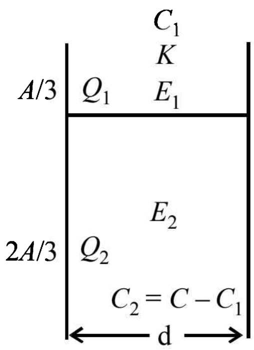
$\therefore$ (c) is correct

$$E_2(r_0/2) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r_0/2} = \frac{1}{4\pi\epsilon_0}\frac{4\lambda}{r_0} = \frac{\lambda}{\pi\epsilon_0 r_0}$$

$$4E_3(r_0/2) = \frac{4\sigma}{2\epsilon_0} = \frac{2\sigma}{\epsilon_0} = \frac{2}{\epsilon_0}\times\frac{\lambda}{\pi r_0}$$

$\therefore$ (d) is incorrect.

27. (a, d)



This is a combination of two capacitors in parallel. Therefore

$C = C_1 + C_2$ $\qquad \therefore C_2 = C - C_1$

where $C_1 = \frac{kA}{3\epsilon_0 d}$ and $C - C_1 = \frac{2A}{3\epsilon_0 d}$

$$\therefore \frac{C - C_1}{C_1} = \frac{2}{k}$$

$$\therefore \frac{C}{C_1} - 1 = \frac{2}{k}$$

$$\therefore \frac{C}{C_1} = \frac{2}{k} + 1$$

$$\frac{C}{C_1} = \frac{2}{k} + 1$$

$\therefore$ (d) is a correct option.

$$\text{Now, } Q_1 = C_1 V = \frac{kA}{3\epsilon_0 d}\times V$$

$$\text{and } Q_2 = (C - C_1)V = \frac{2A}{3\epsilon_0 d}\times V$$

$\therefore \frac{Q_1}{Q_2} = \frac{k}{2}$ $\qquad \therefore$ (c) is incorrect

Also $V = E\times d$

$\therefore E = \frac{V}{d} = E_1 = E_2$ $\qquad \therefore$ (a) is a correct option

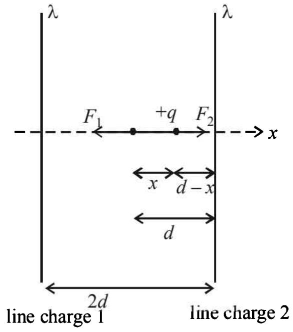
28 (c) Force on change q when it is given a small displacement x is $F_{net} = F_1 - F_2$

$$F_{net} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d - x} - \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d + x}$$

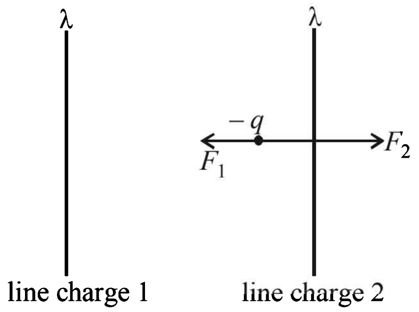
$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\left[\frac{d + x - d + x}{d^2 - x^2}\right]$$

$$\therefore F_{net} = \frac{\lambda}{2\pi\varepsilon_0}\frac{2x}{d^2 - x^2}$$

When $x << d$ then



$F_{net} = \dfrac{\lambda}{\pi\varepsilon_0} x$ and is directed towards the mean position therefore the charge +q will execute SHM.

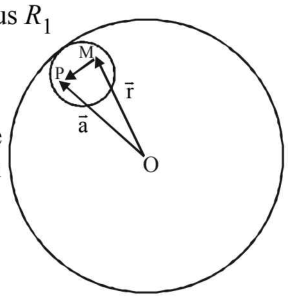


In case of charge $(-q)$

$F_2 > F_1$ therefore the charge $-q$ continues to move in the direction of its displacement.

[C] is the correct option.

29. (d) Assume the cavity to contain similar charge distribution of positive and negative charge as the rest of sphere. Electric field at M due to uniformly distributed charge of the whole sphere of radius R_1



$$\vec{E} = \frac{\rho}{3\varepsilon}\vec{r}$$

Electric field at M due the negative charge distribution in the cavity

$$\vec{E}_2 = \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

∴ The total electric field at M is

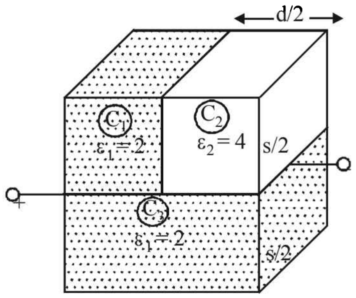
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}\overrightarrow{MP}$$

$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{r} + \frac{\rho}{3\varepsilon}(\vec{a} - \vec{r}) \left[\because \ \vec{r} + \overrightarrow{MP} = \vec{a}\right]$$

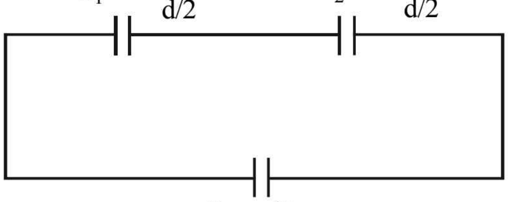
$$\therefore \quad \vec{E} = \frac{\rho}{3\varepsilon}\vec{a}$$

(d) is the correct option

30. (d)



$$C_1 = \frac{2\,\varepsilon_0\, s/2}{d/2} \qquad C_2 = \frac{4\,\varepsilon_0\, s/2}{d/2}$$



$$C_3 = \frac{2\,\varepsilon_0\, s/2}{d} = \frac{\varepsilon_0 s}{d}$$

$$C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2} + C_3 = \frac{\dfrac{2\varepsilon_0 s}{d} \times \dfrac{4\,\varepsilon_0 s}{d}}{\dfrac{6\varepsilon_0 s}{d}} + \frac{\varepsilon_0 s}{d}$$

$$= \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d}$$

$$\therefore \quad C_{eq} = \frac{7}{3}\frac{\varepsilon_0 s}{d} = \frac{7}{3}C_1 \qquad \left[\because C_1 = \frac{\varepsilon_0 s}{d}\right]$$

E. Subjective Problems

1. (i) The force on charge q kept at A due to charges kept at B and C

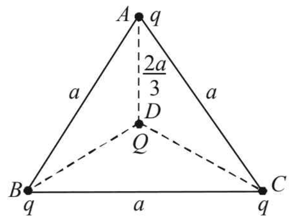
$F_1 = 2F\cos 30°$

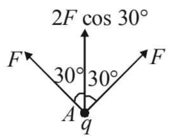
$$F_1 = \sqrt{3} \times \left(9 \times 10^9 \frac{q^2}{a^2}\right)$$

The force on q due to charge $(-q)$ kept at D

$$F_2 = 9 \times 10^9 \frac{q^2}{(2a/3)^2} = \frac{9}{4} \times \left(9 \times 10^9 \times \frac{q^2}{a^2}\right)$$

Clearly the two forces are not equal. Also as $F_2 > F_1$ the charges will move towards the centre.





(ii) For charges to remain stationary

$$2 \times K\frac{q^2}{a^2} \times \frac{\sqrt{3}}{2} = \frac{9}{4} \times K \times \frac{q^2 Q}{a^2} \Rightarrow \frac{4\sqrt{3}\,q}{9} = Q$$

The charge Q should be negative.

The potential energy of the system is

$$= 3\left[K\frac{q^2}{a^2} + K\frac{q^2}{a^2}\right] + 3\left[K \times \frac{4\sqrt{3}}{9}\frac{q \times q}{(2a/3)^2}\right]$$

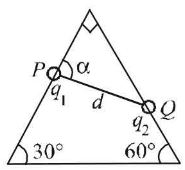
$$= 6K \times \frac{q^2}{a^2} + 3\sqrt{3}\,K\frac{q^2}{a^2} = 3(2 + \sqrt{3})K\frac{q^2}{a^2}$$

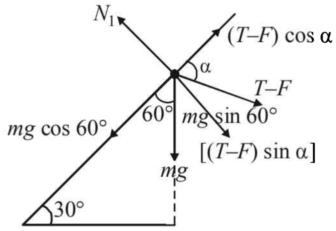
This is the amount of work needed to move the charges to infinity.

2. Because of equilibrium of charge q_1

$N_1 = mg\sin 60° + (T - F)\sin\alpha$ ... (i)

and $(T - F)\cos\alpha = mg\cos 60°$ ... (ii)

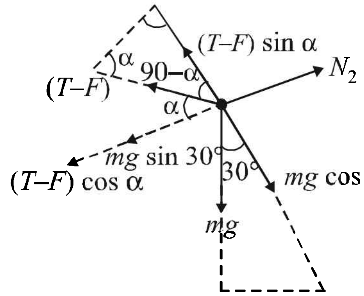




Because of equilibrium of charge q_2

$(T-F)\sin\alpha = mg\cos 30°$ ... (iii)

and $N_2 = (T-F)\cos\alpha + mg\sin 30°$ ... (iv)



From (i) and (iii)

$$N_1 = mg\sin 60° + mg\cos 30°$$

$$= mg\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,mg$$

From (ii) and (iv)

$$N_2 = mg\cos 60° + mg\sin 30° = mg\left(\frac{1}{2}+\frac{1}{2}\right) = mg$$

Also, $F = k\dfrac{q_1q_2}{\ell^2}$

Now from eqn. (ii) and (iii), we get

$(T-F)^2\cos^2\alpha + (T-F)^2\sin^2\alpha$
$= m^2g^2\cos^2 60° + m^2g^2\cos^2 30°$

$$\Rightarrow (T-F)^2 = m^2g^2\left[\frac{1}{4}+\frac{3}{4}\right] = m^2g^2$$

$\Rightarrow T - F = \pm mg$ ... (v)

$\Rightarrow T = mg + F = mg + k\dfrac{q_1q_2}{\ell^2}$ ... (vi)

[Taking positive sign]

From (ii) and (v)

$mg\cos\alpha = mg\cos 60° \Rightarrow \cos\alpha = \cos 60°$

$\therefore \alpha = 60°$

when the string is cut, $T = 0$

$\therefore$ From (vi)

$$mg = \pm k\frac{q_1q_2}{\ell^2} \Rightarrow q_1q_2 = \pm\frac{mg\ell^2}{k}$$

Now the charges should be unlike for equilibrium.

3. Let q be the charge on the inner sphere and $(Q-q)$ be the charge on outer sphere.

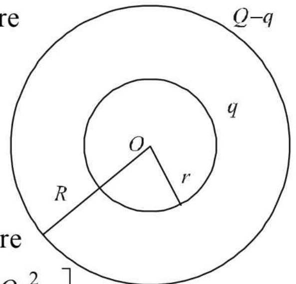
Given that surface charge densities are equal.

$$\therefore \frac{q}{4\pi r^2} = \frac{Q-q}{4\pi R^2} \quad \left(\text{Surface charge density}, \sigma = \frac{q}{A}\right)$$

or, $qR^2 = (Q-q)r^2$ or, $qR^2 = Qr^2 - qr^2$

$$\therefore q = \frac{Qr^2}{R^2+r^2}$$

Potential at O due to inner sphere



$$V_i = K\frac{q}{r} = \frac{K}{r}\left(\frac{Qr^2}{R^2+r^2}\right)$$

$$V_i = K\frac{Qr}{R^2+r^2}$$

Potential at O due to outer sphere

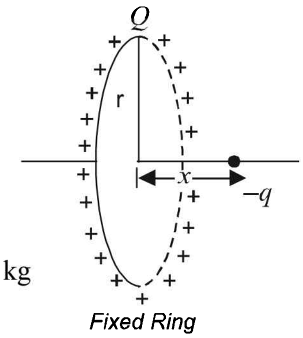
$$V_0 = K\frac{(Q-q)}{R} = \frac{K}{R}\left[Q - \frac{Qr^2}{R^2+r^2}\right]$$

$$= \frac{K}{R}\frac{\left[QR^2+Qr^2-Qr^2\right]}{(R^2+r^2)} = \frac{K(QR)^2}{R\left(R^2+r^2\right)} = \frac{KQR}{\left(R^2+r^2\right)}$$

The total potential at the common centre

$$V = V_i + V_0 = \frac{KQr}{R^2+r^2} + \frac{KQR}{R^2+r^2} = \frac{KQ(R+r)}{R^2+r^2}$$

4. **KEY CONCEPT :** The electric field due a uniformly charged ring of radius r at a point distant x from its center on its axis is given by



$$E = k\frac{Qx}{(r^2+x^2)^{3/2}}$$

$r = 1\text{m}$

$Q = 10^{-5}\,\text{C}$

mass of particle $m = 0.9\times10^{-3}$ kg

charge on particle $q = -10^{-6}$ C

$\therefore$ Force on the negative charge q will be $F = qE$

$$\therefore F = \frac{-kQq}{(r^2+x^2)^{3/2}}\times x \quad \text{or,} \quad mA = \frac{-kQq}{(r^2+x^2)^{3/2}}\times x$$

$$\text{or,} \quad A = -k\frac{Qq}{m(r^2+x^2)^{3/2}}\times x$$

For $x \ll r$ $A = -\dfrac{kQq}{r^3}\times x$

$\Rightarrow$ The motion is simple harmonic in nature.

Comparing the above equation with $A = -\omega^2 x$ we get

$$\therefore \omega^2 = \frac{kQq}{mr^3} \text{ or } \omega = \sqrt{\frac{kQq}{mr^3}}$$

$$\therefore \frac{2\pi}{T} = \sqrt{\frac{kQq}{mr^3}} \Rightarrow T = 2\pi\sqrt{\frac{mr^3}{kQq}}$$

$$T = 2\times3.14\left[\frac{0.9\times10^{-3}\times1^3}{9\times10^9\times10^{-5}\times10^{-6}}\right]^{1/2}$$

$= 6.28\,[0.01]^{1/2} = 6.28\,[0.1]$

$T = 0.628$ sec

5. The potential difference across each capacitor is V.
Total Energy = Energy in A + Energy in B

$$= \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

When the switch opened and a dielectric is inserted between the plates of capacitors, the new capacitance is 3C.

Energy in $A = \frac{1}{2}(3C)V^2 \quad = \frac{3}{2}CV^2$ (V is the same)

Energy in $B = \frac{1}{2}\frac{q^2}{KC} = \frac{1}{2} \times \frac{(CV)^2}{3C}$

$= \frac{CV^2}{6}$ (charge on capacitor B remains same when switch is opened)

Total Energy = Energy in A + Energy in B

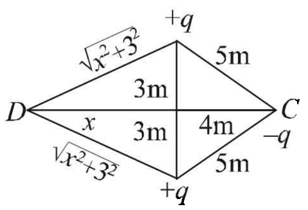
$\therefore$ Total Energy $= \frac{3}{2}CV^2 + \frac{1}{6}CV^2 = \frac{5}{3}CV^2$ ... (ii)

$$\frac{\text{Total Energy initially}}{\text{Total energy finally}} = \frac{CV^2}{\frac{5}{3}CV^2} = \frac{3}{5}$$

6. Total energy of the system of three charges when the charge $-q$ is at C

= P.E. + K.E.

$$= \left[\frac{Kq \times q}{6} + \frac{K(q)(-q)}{5} + \frac{Kq(-q)}{5}\right] + 4 \quad \text{... (i)}$$



Final energy of the system of three charges when $-q$ is at D and momentarily at rest

= P.E. + K.E.

$$= \left[\frac{Kq \times q}{6} + \frac{Kq(-q)}{\sqrt{x^2+3^2}} + \frac{Kq(-q)}{\sqrt{x^2+3^2}}\right] + 0$$

$$= \frac{Kq \times q}{6} + \frac{2Kq(-q)}{\sqrt{x^2+3^2}} \quad \text{... (ii)}$$

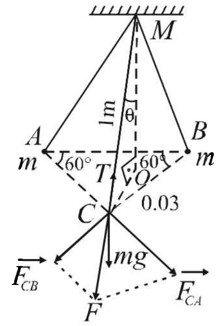
By the principle of conservation of energy from (i) and (ii), we get

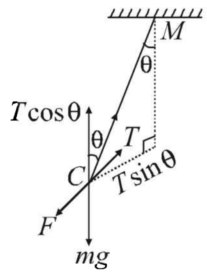
$$\frac{kq \times q}{6} + \frac{2kq(-q)}{5} + 4 = \frac{kq \times q}{6} + \frac{2kq(-q)}{\sqrt{x^2+3^2}}$$

$$2 = kq^2\left[\frac{1}{5} - \frac{1}{\sqrt{x^2+3^2}}\right]$$

$\therefore$ $x^2 + 9 = 81$ $\therefore x = 8.48$ m

7. Each mass will be in equilibrium under the act of three force namely tension of string, weight, resultant electrostatic force of the two other charges out of these three forces F and mg are perpendicular.





Let T make an angle θ with the vertical

$OC = \frac{2}{3}\sqrt{(0.03)^2 - (0.015)^2} = 0.0173$ m

$\therefore$ $OM = 0.9997$

NOTE THIS STEP : Resolving T in the direction of mg and F and applying the condition of equilibrium, we get

$T\cos\theta = mg$; $\quad T\sin\theta = F$

$$\therefore \tan\theta = \frac{F}{mg} \quad \text{... (i)}$$

$$F = \sqrt{F_{CA}^2 + F_{CB}^2 + 2F_{CA}F_{CB}\cos\alpha}$$

$$\therefore F = \sqrt{F_{CA}^2 + F_{CA}^2 + 2F_{CA}^2 \times \frac{1}{2}}$$

$$F = \sqrt{3}F_{CA} = \sqrt{3} \times \frac{kq^2}{(CA)^2} \quad \text{... (ii)}$$

[where F_{CB} = Force on C due to B
F_{CA} = Force on C due to A
$|\vec{F}_{CB}| = |\vec{F}_{CA}|$ and $\alpha = 60°$]

Also, $\tan\theta = \frac{OC}{OM} = \frac{0.0173}{0.9997}$ .... (iii)

From (i), (ii) and (iii)

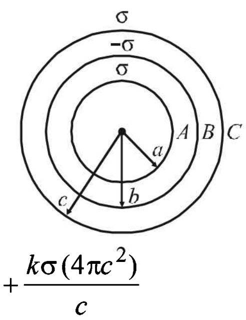
$$\frac{0.0173}{0.9997} = \frac{\sqrt{3} \times 9 \times 10^9 \times q^2}{(0.03)^2 \times 10^{-3} \times 9.8}$$

On solving, we get $q = 3.16 \times 10^{-9}$ C.

8. Time for the dipole to align along the direction of electric field will be

$$t = \frac{T}{4} = \frac{2\pi}{4}\sqrt{\frac{ML}{2qE}} = \frac{\pi}{2}\sqrt{\frac{ML}{2qE}}.$$

9. Charge on Shell $A = q_A = \sigma(4\pi a^2)$
Charge on Shell $B = q_B = \sigma(4\pi b^2)$
Charge on Shell $C = q_C = \sigma(4\pi c^2)$
The potential of shell A



$$V_A = \frac{kq_A}{a} + \frac{kq_B}{b} + \frac{kq_C}{c}$$

$$= \frac{k\sigma(4\pi a^2)}{a} + \frac{k(-\sigma)(4\pi b^2)}{b} + \frac{k\sigma(4\pi c^2)}{c}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \sigma \times \frac{4\pi a^2}{a} - \frac{1}{4\pi\varepsilon_0}\sigma\frac{(4\pi b^2)}{b} + \frac{1}{4\pi\varepsilon_0} \times \sigma \frac{(4\pi c^2)}{c}$$

$$= \frac{\sigma}{\varepsilon_0}[a-b+c] \text{ Similarly, } \quad V_B = \frac{kq_A}{b} + \frac{kq_B}{b} + \frac{kq_C}{c}$$

and $V_C = \frac{kq_A}{c} + \frac{kq_B}{c} + \frac{kq_C}{c}$

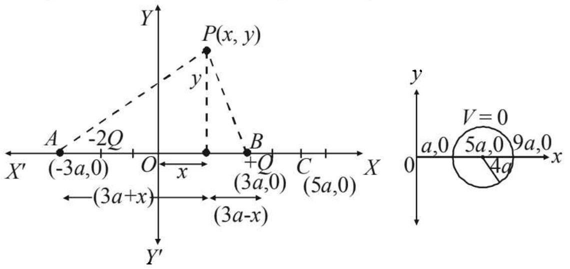
$$V_B = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right] \text{ and } V_C = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2+c^2}{c}\right]$$

Given that $V_A = V_C$

$$\frac{\sigma}{\varepsilon_0}(a-b+c) = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2+c^2}{c}\right]$$

or $\quad ac - bc + c^2 = a^2 - b^2 + c^2 \quad$ or $c = a + b$

10. **(a)** Let P be a point in the X-Y plane with coordinates (x, y) at which the potential due to charges $-2Q$ and $+Q$ placed at A and B respectively be zero.



$$\therefore \quad \frac{K(2Q)}{\sqrt{(3a+x)^2+y^2}} = \frac{K(+Q)}{\sqrt{(3a-x)^2+y^2}}$$

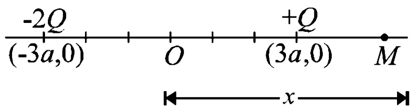
$$\Rightarrow \quad 2\sqrt{(3a-x)^2+y^2} = \sqrt{(3a+x)^2+y^2}$$

$$\Rightarrow \quad (x-5a)^2 + (y-0)^2 = (4a)^2$$

This is the equation of a circle with centre at $(5a, 0)$ and radius $4a$. Thus C $(5a, 0)$ is the centre of the circle.

(b) For $x > 3a$

To find $V(x)$ at any point on X-axis, let us consider a point (arbitrary) M at a distance x from the origin.



The potential at M will be

$$V(x) = \frac{K(-2Q)}{x+3a} + \frac{K(+Q)}{(x-3a)} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore \quad V(x) = KQ\left[\frac{1}{x-3a} - \frac{2}{x+3a}\right] \text{ for } |x| > 3a$$

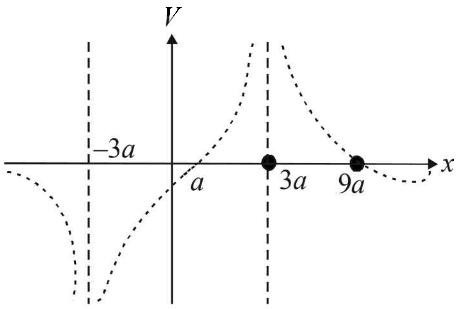
Similarly, for $0 < |x| < 3a$

$$V(x) = KQ\left[\frac{1}{3a-x} - \frac{2}{3a+x}\right]$$

Since circle of zero potential cuts the x-axis at $(a,0)$ and $(9a,0)$

Hence, $V(x) = 0$ at $x = a$, at $x = 9a$

- From the above expressions
  $V(x) \to \infty$ at $x \to 3a$ and $V(x) \to -\infty$ at $x \to -3a$
- $V(x) \to 0$ as $x \to \pm\infty$
- $V(x)$ varies $\frac{1}{x}$ in general.



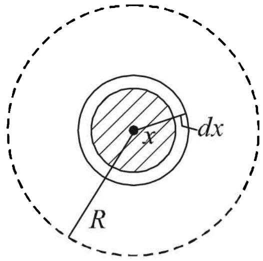
(c) Applying Energy Conservation

$(\text{K.E.} + \text{P.E.})_{\text{centre}} = (\text{K.E.} + \text{P.E.})_{\text{circumference}}$

$$0 + K\left[\frac{Qq}{2a} - \frac{2Qq}{8a}\right] = \frac{1}{2}mv^2 + K\left[\frac{Qq}{6a} - \frac{2Qq}{12a}\right]$$

$$\frac{1}{2}mv^2 = \frac{KQq}{4a}, \quad v = \sqrt{\frac{KQq}{2ma}} = \sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$$

11. **(a)** Let us consider a shell of the thickness dx at a distance x from the centre of a sphere



$$\text{The vol. of the shell} = \frac{4}{3}\pi\left[(x+dx)^3 - \frac{4}{3}\pi x^3\right]$$

$$= \frac{4}{3}\pi\left[(x+dx)^3 - x^3\right]$$

$$= \frac{4}{3}\pi x^3\left[\left(1+\frac{dx}{x}\right)^3 - 1\right]$$

$$= \frac{4}{3}\pi x^3\left[1 + \frac{3dx}{x} - 1\right]$$

$$= \frac{4}{3}\pi x^3 \times \frac{3dx}{x} = 4\pi x^2 dx$$

Let ρ be the charge per unit volume of the sphere

$\therefore \quad$ Charge of the shell $= dq = 4\pi x^2 \rho dx \qquad$ ... (i)

Potential at the surface of the sphere of radius x

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \times \frac{4}{3}\pi x^3}{x} \qquad \left[\because V = k\frac{q}{r}\right]$$

$\therefore$ Potential at the surface of the sphere of radius x =

$$\frac{\rho x^2}{3\varepsilon_0}$$

Work done in bringing the charge dq on the sphere of radius x

$$dW = \frac{\rho x^2}{3\varepsilon_0} \times dq \Rightarrow dW = \frac{\rho x^2}{3\varepsilon_0} \times 4\pi x^2 \rho\, dx$$

Therefore the work done in accumulating the charge Q over a spherical volume of radius R meters

$$W = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0} x^4 dx = \frac{4\pi\rho^2}{3\varepsilon_0}\left[\frac{x^5}{5}\right]_0^R = \frac{4\pi\rho^2}{3\varepsilon_0}\frac{R^5}{5}$$

$$= \frac{4\pi}{3\varepsilon_0}\left(\frac{Q}{4/3\pi R^3}\right)^2 \frac{R^5}{5} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

This is also the energy stored in the system.

(b) The above energy calculated is

$$E = \frac{3Q^2}{5\times(4\pi\varepsilon_0)R} = \frac{3KQ^2}{5R} \text{ where } K = \frac{1}{4\pi\varepsilon_0}$$

NOTE : In case of earth and gravitational pull, K may be replaced by G. Therefore the energy required to disassemble the planet earth against the gravitational pull amongst its constituent particle is the work required to make earth from its constituent particles.

$$\therefore \quad E = \frac{3GM^2}{5R} \qquad [\because Q \text{ is replaced by } M]$$

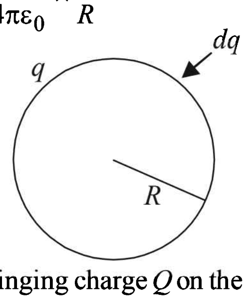
But $g = \dfrac{GM}{R^2} \Rightarrow gMR = \dfrac{GM^2}{R}$

$$F = \frac{Kq_1q_2}{r^2};\ F = \frac{Gm_1m_2}{r^2}$$

$$\therefore \quad E = \frac{3}{5}gMR = \frac{3}{5}\times 10\times 2.5\times 10^{31} = 1.5\times 10^{32}\text{ J}$$

(c) During the charging process, let at any instant the spherical conductor has a charge q on its surface.

The potential at the surface $= \dfrac{1}{4\pi\varepsilon_0}\times\dfrac{q}{R}$



Small amount of work done in increasing charge dq more on the surface will be

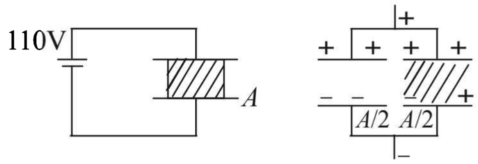
$$dW = \frac{1}{4\pi\varepsilon_0}\times\frac{q}{R}\times dq$$

$\therefore$ Total amount of work done in bringing charge Q on the surface of spherical conductor.

$$W = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q q\,dq = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{(8\pi\varepsilon_0 R)}$$

12. **(i) NOTE :** The capacitor A with dielectric slab can be considered as two capacitors in parallel, one having dielectric slab and one not having dielectric slab. Each capacitor has an area of $\dfrac{A}{2}$.

The combined capacitance is



$$C = C_1 + C_2$$

$$= \frac{(A/2)\varepsilon_0}{d} + \frac{(A/2)\varepsilon_0\varepsilon_r}{d} = \frac{A}{2}\frac{\varepsilon_0}{d}[1+\varepsilon_r]$$

$$= \frac{0.4\times 8.85\times 10^{-12}}{2\times 8.85\times 10^{-4}}[1+9] = 2\times 10^{-9} F$$

$$\therefore \quad \text{Energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}\times 2\times 10^{-9}\times(110)^2$$

$$= 1.21\times 10^{-5}\text{ J}$$

(ii) Work done in removing the dielectric slab = (Energy stored in capacitor without dielectric) – (Energy stored in capacitor with dielectric).

NOTE : While taking out the dielectric, the charge on the capacitor plate remains the same.

$$\therefore \quad W = \frac{q^2}{2C'} - \frac{q^2}{2C} \quad \text{Here, } C = 2\times 10^{-9} F,$$

$$C' = \frac{A\varepsilon_0}{d} = \frac{0.04\times 8.85\times 10^{-14}}{8.85\times 10^{-4}} = 0.4\times 10^{-9} F$$

$$q = CV = 2\times 10^{-9}\times 110 = 2.2\times 10^{-7} C$$

$$\therefore \quad W = \frac{(2.2\times 10^{-7})^2}{2}\left[\frac{1}{0.4\times 10^{-9}} - \frac{1}{2\times 10^{-9}}\right]$$

$$= 4.84\times 10^{-5}\text{ J}$$

(iii) The capacitance of $B = \dfrac{\varepsilon_0\varepsilon_r A_B}{d}$

$$= \frac{8.85\times 10^{-12}\times 9\times 0.02}{8.85\times 10^{-4}}$$

$$C_B = 1.8\times 10^{-9} F$$

The charge on A, $q_A = 2.2\times 10^{-7} C$ gets distributed into two parts.

$\therefore \quad q_1 + q_2 = 2.2\times 10^{-7} C$

also the potential difference across A = p.d. across B

$$\frac{q_1}{C_A} = \frac{q_2}{C_B}$$

$$\Rightarrow q_1 = \frac{C_A}{C_B}q_2 = \frac{0.4\times 10^{-9}}{1.8\times 10^{-9}}q_2 = 0.22\,q_2$$

$$\therefore \quad 0.22\,q_2 + q_2 = 2.2\times 10^{-7}$$

$$\Rightarrow q_2 = \frac{2.2}{1.22}\times 10^{-7} = 1.8\times 10^{-7} C$$

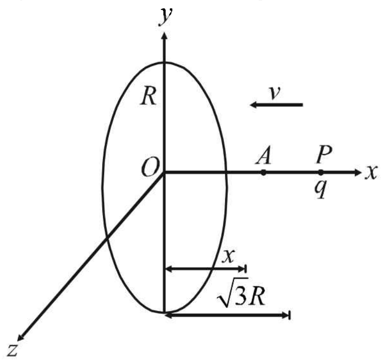
$$\Rightarrow q_1 = 0.4\times 10^{-7} C$$

Total energy stored $= \dfrac{q_1^2}{2C_A} + \dfrac{q_2^2}{2C_B}$

$$= \frac{0.4 \times 0.4 \times 10^{-14}}{2 \times 0.4 \times 10^{-9}} + \frac{1.8 \times 1.8 \times 10^{-14}}{2 \times 1.8 \times 10^{-8}}$$

$= 0.2 \times 10^{-5} + 0.9 \times 10^{-5} = 1.1 \times 10^{-5}$ J

13. Potential energy can be found at the initial point A and final point O. The difference in potential energy has to be provided by the K.E. of the charge at A.



$$V(x) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\sqrt{R^2 + x^2}}, \text{ at } A.$$

$$V_O = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi R\lambda}{R}, \text{ at } O. \text{ or } V_O = \frac{\lambda}{2\varepsilon_0}$$

$$V_P = \frac{1}{4\pi\varepsilon_0} \frac{2\pi R\lambda}{\sqrt{R^2 + (\sqrt{3}R)^2}} = \frac{\lambda}{4\varepsilon_0}$$

Potential difference between points O and $P = V$

$\therefore \quad V = V_O - V_P$

or $\quad V = \dfrac{\lambda}{2\varepsilon_0} - \dfrac{\lambda}{4\varepsilon_0}$ or $V = \dfrac{\lambda}{4\varepsilon_0}$

The kinetic energy of the charged particle is converted into its potential energy at O.

$\therefore$ Potential energy of charge $(q) = qV$

Kinetic energy of charged particle $= \dfrac{1}{2}mv^2$

For minimum speed of particle so that it does not return to P,

$$\frac{1}{2}mv^2 = qV \text{ or } v^2 = \frac{2qV}{m} = \frac{2q \times \lambda}{m \times 4\varepsilon_0}$$

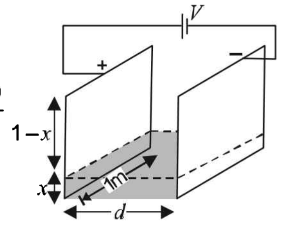
or $\quad v = \sqrt{\dfrac{q\lambda}{2\varepsilon_0 m}}$

14. The adjacent figure is a case of parallel plate capacitor. The combined capacitance will be

$C = C_1 + C_2$

$$= \frac{k\varepsilon_0(x \times 1)}{d} + \frac{\varepsilon_0[(1-x) \times 1]}{d}$$

$$C = \frac{\varepsilon_0}{d}[kx + 1 - x] \qquad \text{...(i)}$$



Differentiating the above quation w.r.t. time

$$\frac{dC}{dt} = \frac{\varepsilon_0}{d}(k-1)\frac{dx}{dt} = \frac{\varepsilon_0}{d}(k-1)v$$

where $\quad v = \dfrac{dx}{dt}$

We know that $q = CV$, $\dfrac{dq}{dt} = V\dfrac{dC}{dt}$

From (iii) and (iv)

$$I = V\frac{\varepsilon_0}{d}(k-1)v$$

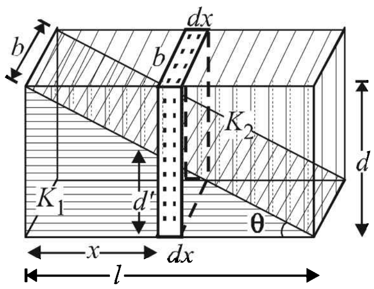
$$I = \frac{500 \times 8.85 \times 10^{12}}{0.01}(11-1) \times 0.001$$

$= 4.425 \times 10^{-9}$ Amp.

15. **Case (i)** When no dielectric :

Given $\quad C = \dfrac{\varepsilon_0 A}{d}$

Case (ii) When dielectric is filled : A small dotted element of thickness dx is considered as shown in the figure.



The small capacitance of the dotted portion

$\dfrac{1}{dC} = \dfrac{1}{dC_1} + \dfrac{1}{dC_2}$ where dC_1 = capacitance of capacitor with dielectric K_1

dC_2 = capacitance of capacitor with dielectric K_2.

Let ℓ, b the length and breadth of the capacitor plate. Therefore $\ell \times b = A$.

$$dC_1 = \frac{K_1(bdx)\varepsilon_0}{d'}$$

$$d' = d - x\frac{d}{\ell} = d\left[1 - \frac{x}{\ell}\right]$$

$$\therefore \quad dC_1 = \frac{K_1 b(dx)\,\varepsilon_0}{d\left[1 - \frac{x}{\ell}\right]} - \frac{K_1 b\ell(dx)\,\varepsilon_0}{d(\ell - x)} = \frac{K_1 A\,\varepsilon_0(dx)}{d(\ell - x)}$$

Similarly, $dC_2 = \dfrac{K_2\,\varepsilon_0(bdx)}{d - d'} = \dfrac{K_2\,\varepsilon_0 bdx}{d - d + \frac{xd}{\ell}}$

$$\frac{K_2\varepsilon_0 b.\ell.dx}{xd} = \frac{K_2\,\varepsilon_0 A\,dx}{xd}$$

$$\therefore \quad \frac{1}{dC}=\frac{d(\ell-x)}{K_1A\varepsilon_0(dx)}+\frac{xd}{K_2A\varepsilon_0(dx)}$$

$$\Rightarrow \quad \frac{K_1K_2A\varepsilon_0 dx}{K_2\ell d+d(K_1-K_2)x}=dC$$

To find the capacitance of the whole capacitor, we integrate the above equation.

$$C=\int_0^{\ell}\frac{K_1K_2A\varepsilon_0 dx}{K_2\ell d+d(K_1-K_2)x}$$

$$=K_1K_2A\varepsilon_0\int_0^{\ell}\frac{dx}{K_2\ell d+d(K_1-K_2)x}$$

$$=K_1K_2A\varepsilon_0\left[\frac{\log[K_2\ell d+d(K_1-K_2)x]}{d(K_1-K_2)}\right]_0^{\ell}$$

$$C=\frac{K_1K_2A\varepsilon_0}{d(K_1-K_2)}\log\frac{K_1}{K_2}$$

16. **(i)** **KEY CONCEPT :** Use charge conservation to solve this problem.

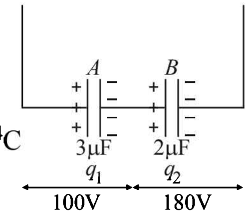
INITIALLY:

Charge on capacitor A

$q_A=3\times10^{-6}\times100=3\times10^{-4}$C

Charge on capacitor B

$q_B=2\times10^{-6}\times180=3.6\times10^{-4}$C

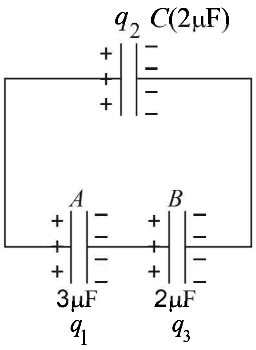


FINALLY:

Let the charge on capacitor A, C and B be q_1, q_2 and q_3 respectively.

By charge conservation.

The sum of charge on +ve plate of capacitor A and C should be equal to q_A



$\therefore \quad q_1+q_2=3\times10^{-4}$C... (i)

Similarly the sum of charge on –ve plates of capacitor C and B will be equal to q_B

$\therefore \quad -q_2-q_3=-3.6\times10^{-4}$C

$\Rightarrow \quad q_2+q_3=3.6\times10^{-4}$C ... (ii)

Applying Kirchoff's law in the closed loop, we get

$$\frac{q_1}{3\times10^{-6}}-\frac{q_2}{2\times10^{-6}}+\frac{q_3}{2\times10^{-6}}=0$$

$\Rightarrow \quad 2q_1-3q_2+3q_3=0$ ....(iii)

On solving (i), (ii) and (iii), we get

$q_1=90\times10^{-6}$C, $q_2=210\times10^{-6}$C,

and $q_3=150\times10^{-6}$C,

(ii) Amount of electrostatic energy in the system initially

$$U_i=U_A+U_B=\frac{1}{2}C_A(V_A)^2+\frac{1}{2}C_B(V_B)^2$$

$$=\frac{1}{2}\times3\times10^{-6}(100)^2+\frac{1}{2}\times2\times10^{-6}(180)^2$$

$=4.74\times10^{-2}$J

Amount of electrostatic energy stored finally

$$U_f=\frac{1}{2}\frac{q_1^2}{C_A}+\frac{1}{2}\frac{q_2^2}{C_B}+\frac{1}{2}\frac{q_3^2}{C_C}$$

$$=\frac{1}{2}\frac{(90\times10^{-6})^2}{3\times10^{-6}}+\frac{1}{2}\frac{(210\times10^{-6})^2}{2\times10^{-6}}+\frac{1}{2}\frac{(150\times10^{-6})^2}{2\times10^{-6}}$$

$$+\frac{1}{2}\frac{(150\times10^{-6})^2}{2\times10^{-6}}=1.8\times10^{-2}\text{ J}$$

17. Limiting value of energy as $n\to\infty$.

Let us calculate q_n when n tends to ∞.

For G.P., $S_\infty=\dfrac{a}{1-r_1}$ where r_1 = common ratio

$$\therefore \quad q_\infty=\frac{QR}{R+r}\left[\frac{1}{1-\frac{R}{R+r}}\right]\text{ or } q_\infty=\frac{QR}{r}$$

$$\therefore \quad U_\infty=\frac{q_\infty^2}{2C}=\left(\frac{QR}{r}\right)^2\times\frac{1}{2\times(4\pi\varepsilon_0)\times(R)}$$

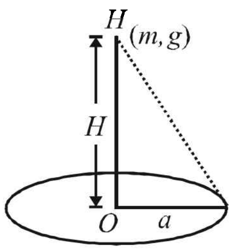
$$\text{or} \quad U_\infty=\frac{Q^2R^2}{r^2\times2\times4\pi\varepsilon_0R}\text{ or } U_\infty=\frac{Q^2R}{2(4\pi\varepsilon_0)r^2}$$

18. **(a)** **KEY CONCEPT :** The K.E. of the particle, when it reaches the disc is zero.

Given that a = radius of disc, σ = surface charge density, $q/m=4\varepsilon_0 g/\sigma$

Potential due to a charged disc at any axial point situated at a distance x from O is,

$$V(x)=\frac{\sigma}{2\varepsilon_0}[\sqrt{a^2+x^2}-x]$$



Hence, $V(H)=\dfrac{\sigma}{2\varepsilon_0}[\sqrt{a^2+H^2}-H]$

and $V(O)=\dfrac{\sigma a}{2\varepsilon_0}$

NOTE : According to law of conservation of energy, loss of gravitational potential energy = gain in electric potential energy

$mgH=q\Delta V$

$=q[V(0)-V(H)]$

$$mgH=q\frac{\sigma}{2\varepsilon_0}[a-\{\sqrt{(a^2+H^2)}-H\}] \quad ...(1)$$

From the given relation : $\dfrac{\sigma q}{2\varepsilon_0}=2mg$

Putting this in equation (1), we get,

$$mgH=2mg[a-\{\sqrt{(a^2+H^2)}-H\}]$$

or $H=\dfrac{4a}{3}$ $\quad$ [$\because H=O$ is not valid]

(b) Total potential energy of the particle at height H

$U(x) = mgx + qV(x)$

$$= mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2 + x^2} - x)$$

$$= mgx + 2mg\,[\sqrt{(a^2 + x^2)} - x] \qquad ....(2)$$

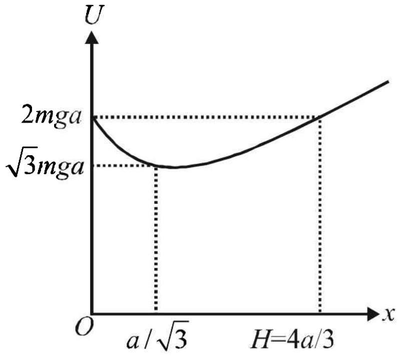
$$U_{(A)} = mgH + 2mg\left[\sqrt{a^2 + H^2} - H\right]$$

$$= mg\left[2\sqrt{a^2 + H^2} - H^2\right] \qquad ....(3)$$

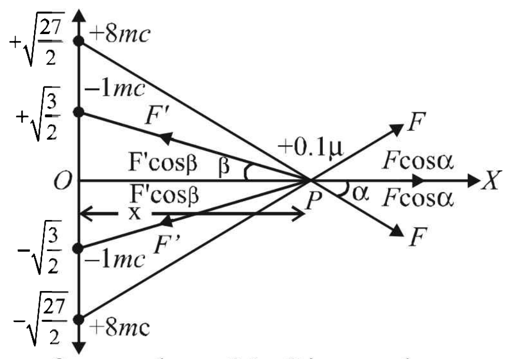
For equilibrium : $\frac{dU}{dH} = 0$

This gives : $H = \frac{a}{\sqrt{3}}$ $\therefore$ $U_{min} = \sqrt{3}$ mga

From equation (2), graph between $U(x)$ and x is as shown above.



19. Let the particle at some instant be at a point P distant x from the origin. As shown in the figure, there are two forces of repulsion acting due to two charges of + 8 mC. The net force is $2F\cos\alpha$ towards right.
Similarly there are two forces of attraction due to two charges of – 1 mC. The net force due to these force is $2F'\cos\beta$ towards left.



The net force on charge 0.1 μC is zero when

$2F\cos\alpha = 2F'\cos\beta$

$$\frac{K \times 8 \times 10^{-6} \times 0.1 \times 10^{-6}}{\left(\sqrt{x^2 + \frac{27}{2}}\right)^2} \times \frac{x}{\sqrt{x^2 + \frac{27}{2}}}$$

$$= \frac{K \times 1 \times 10^{-6} \times 0.1 \times 10^{-6}}{\left(\sqrt{x^2 + \frac{3}{2}}\right)^2} \times \frac{x}{\sqrt{x^2 + \frac{3}{2}}}$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{5}{2}}$$

This means that we need to move the charge from $-\infty$ to $\sqrt{\frac{5}{2}}$. Thereafter the attractive forces will make the charge move to origin.

The electric potential of the four charges at $x = \sqrt{\frac{5}{2}}$ is

$$V = \frac{2 \times 9 \times 10^9 \times 8 \times 10^{-6}}{\sqrt{\frac{5}{2} + \frac{27}{2}}} - \frac{2 \times 9 \times 10^9 \times 10^{-6}}{\sqrt{\frac{5}{2} + \frac{3}{2}}}$$

$$= 2 \times 9 \times 10^9 \times 10^{-6}\left[\frac{8}{4} - \frac{1}{2}\right] = 2.7 \times 10^4\,\text{V}$$

Kinetic energy is required to overcome the force of repulsion from ∞ to $x = \sqrt{\frac{5}{2}}$.

The work done in this process is $W = q\,(V)$

where V = p.d between ∞ and $x = \sqrt{\frac{5}{2}}$.

$\therefore$ $W = 0.1 \times 10^{-6} \times 2.7 \times 10^4 = 2.7 \times 10^{-3}$ J

By energy conservation $\frac{1}{2}mV_0^2 = 2.7 \times 10^{-3}$

$$\Rightarrow \quad \frac{1}{2} \times 6 \times 10^{-4} V_0^2 = 2.7 \times 10^{-3}$$

$\Rightarrow$ $V_0 = 3$ m/s

K.E. at the origin

Potential at origin

$$V_{x=0} = \frac{2 \times 9 \times 10^9 \times 8 \times 10^{-6}}{\sqrt{\frac{27}{2}}} - \frac{2 \times 9 \times 10^9 \times 10^{-6}}{\sqrt{\frac{3}{2}}}$$

$= 2.4 \times 10^4$

Again by energy conservation

$$\text{K.E.} = q\left[V_{x=\frac{\sqrt{5}}{2}} - V_{x=0}\right]$$

$\therefore$ K.E. $= 0.1 \times 10^{-6}\,[2.7 \times 10^4 - 2.4 \times 10^4]$

$= 0.1 \times 10^{-6} \times 0.3 \times 10^4$

$= 3 \times 10^{-4}$ J

20. $$W_{\text{external}} = \Delta PE = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[\frac{-3}{1} + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right] \times 4$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}.\frac{4}{\sqrt{6}}[3\sqrt{3} - 3\sqrt{6} - \sqrt{2}]$$

21. **(a)** Potential energy of the dipole-charge system

$U_i = 0$ (since the charge is far away)

$$U_f = -Q \times \frac{1}{4\pi\varepsilon_0}\frac{p}{d^2} \quad \text{[at a point } (d, \omega)]$$

$\therefore$ $\text{K.E.} = |U_f - U_i| = \frac{1}{4\pi\varepsilon_0}\frac{pQ}{d^2}$

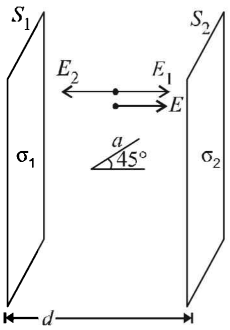
(b) Electric field at origin due to dipole

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{i}$$

Thus, force on charge Q is given by

$$\vec{F} = Q\vec{E} = \frac{2pQ}{4\pi\varepsilon_0 d^3}\hat{i}$$

22. Electric field due to S_1, $E_1 = \frac{\sigma_1}{\varepsilon_0}$



Electric field due to S_2, $E_2 = \frac{\sigma_2}{\varepsilon_0}$

$\therefore$ $E = E_1 - E_2$

$= \frac{\sigma_1 - \sigma_2}{\varepsilon_0} \quad (\because \sigma_1 > \sigma_2)$

Work done by electric field

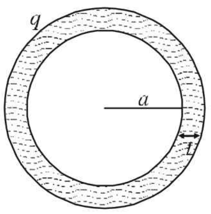
$$W = (q_0 E) a\cos 45^\circ = q_0 E \times \frac{a}{\sqrt{2}}$$

$\therefore$ $W = \frac{q_0(\sigma_1 - \sigma_2)a}{\sqrt{2}\,\varepsilon_0}$

23. *LIQUID BUBBLE :* The potential of the liquid bubble is V.

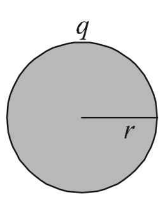
$\Rightarrow$ $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$ ...(1)

where q is the charge on the liquid bubble.



LIQUID DROPLET

The volume of liquid droplet = Volume (of the liquid) in liquid bubble.



$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(a+t)^3 - \frac{4}{3}\pi a^3$$

or, $r^3 = a^3 + t^3 + 3a^2t + 3at^2 - a^3$

or, $r^3 = 3a^2t$

($\because$ t is very small as compared to a)

or, $r = [3a^2t]^{1/3}$ ...(iii)

NOTE : By charge conservation we can conclude that charge on liquid bubble is equal to charge on liquid droplet

Let charge on liquid droplet is q.

$\therefore$ Potential on liquid droplet

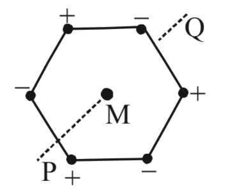
$$V_{\text{droplet}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

or, $V_{\text{droplet}} = \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi\varepsilon_0 V \times a}{[3a^2t]^{1/3}}$ [From (i) and (ii)]

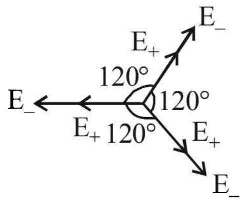
or, $V_{\text{droplet}} = V\left[\frac{a}{3t}\right]^{1/3}$

F. Match the Following

1.



The electric field at M due to the charges at the corners of regular hexagon is as shown



Here $|E_+| = |E_-|$. The symmetry of the situation shows that $E = 0$ at M.

Therefore (A) is the correct option.

The electric potential due to all the charges at M is zero.

Therefore (B) is incorrect option.

When the system of charges is rotated about line PM, the net current will be zero.

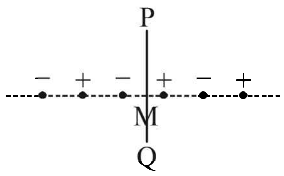
Therefore the magnetic field at M is zero.

(C) is the correct option.

When magnetic field is zero, then $\mu = 0$

(D) is incorrect option.

(q)



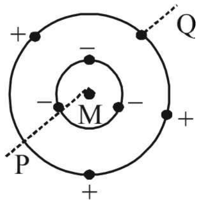
The electric field due to the inner most positive and negative charges at M is $E_1 = 2\left[k\frac{q}{r^2}\right]$ towards left. The electic field due to the next positive and negative charges at M is $E_2 = 2\left[k\frac{q}{(2r)^2}\right]$ towards right. The electric field due to the outermost positive and negative charges at M is $E_3 = 2\left[k\frac{q}{(3r)^2}\right]$ towards left. Clearly the vector sum of these three electric field is not zero.

(A) is incorrect option.

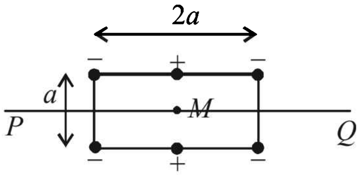
The electric potential due to the charges at M

$$= k\left[\frac{+q}{r} \frac{-q}{r} + \frac{q}{2r} - \frac{q}{2r} + \frac{q}{3r} - \frac{q}{3r}\right] = 0$$

(B) is incorrect option.
The net current due to the innermost positive and negative charges is zero. Similarly the net current due to other charges in pairs is zero. Therefore the magnetic field at M is zero. Also the magnetic moment is zero.
(C) is the correct option
(D) is incorrect option.
(r)
The net electric field due to negative charges in the inner circle is zero. Similarly the net electric field due to positive charges in the outercircle is zero.



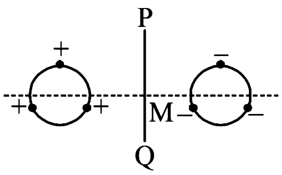
(A) is the correct option.
The electric potential due to negative charges at M is different from the electric potential due to positive charges at M. Therefore the electric potential at M is not equal to zero.
(B) is the correct option.
When the system of charges rotate, we get a current I_1 due to negative charges and another current I due to positive charges. The magnitude of the magnetic at M due to the currents is different. Therefore $B \neq 0$ and $\mu \neq 0$.
(C) is incorrect option
(D) is the correct option.
(s)



The electric field at M due to all the charges is zero because the electric field due to different charges cancel out in pairs.
(A) is the correct option.
The potential at M due to the charges is

$$V = k\left[\frac{+q}{a/2} + \frac{q}{a/2} - 4\left(\frac{q}{\frac{\sqrt{5}a}{4}}\right)\right] \neq 0$$

(B) is the correct option.
When the whole system is set into rotation with a constant angular velocity about the line PQ we get three loops in which current is flowing.
The magnetic field due to these currents produce a resultant magneic field at M which is not equal to zero. Therefore a net magnetic dipole moment will be produced.
(C) is an incorrect option.
(D) is correct option.

(t)


There will be a net electric field due to the arrangement of charges at M towards the right side.
(A) is an incorrect option.
The electic potential at M will can out in pairs by positive and negative charges, due to symmetrical arrangement of charges.
(B) is an incorrect option.

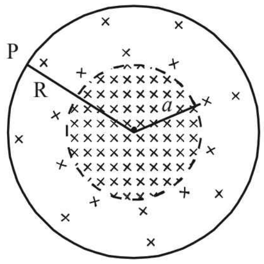
When the system of charges rotates about PQ, the net current is zero due to symmetrical arrangement of charges. Therefore $B = 0$ and $\mu = 0$
(C) is the correct option.
(D) is the incorrect option.

18. (a) If Q_1, Q_2, Q_3 and Q_4 are all positive, then the force will be along $+y$-direction.
If Q_1, Q_2 are positive and Q_3, Q_4 are negative the force will act along $+x$-direction.
If Q_1, Q_4 are positive and Q_2, Q_3 are negative then attractive force will dominate repulsive force and the force will be along $-y$ direction.

G. Comprehension Based Questions

1. (a) When the point of observation is on the surface of sphere then the whole charge inside the sphere (when distributed symmetrically about the centre) behaves as a point charge on the centre. Therefore until the charge distribution is symmetrical about the centre it does not matter what is the ratio a/R. The electric field remains constant and is equal to

$$E = \frac{1}{4\pi \epsilon_0} \cdot \frac{Ze}{R}.$$



2. (b) For a = 0, the graph is as shown. The equation for the graph line is

$$\rho = d - \frac{r}{R} dr$$

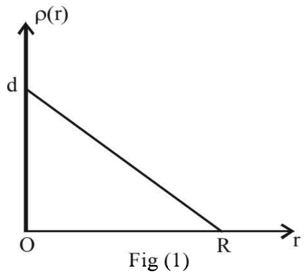


Fig (1)

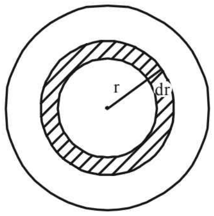


Fig (2)

The charge in the dotted element shown in Fig (2) is
$dq = \rho \times 4\pi r^2 dr$

$$\therefore dq = \left(d - \frac{d}{R} r\right) 4\pi r^2 dr \Rightarrow Ze = \int_0^R 4\pi d r^2 dr - \int_0^R \frac{4\pi d}{R} r^3 dr$$

$$Ze = 4\pi d \frac{R^3}{3} - \frac{4\pi d}{R} \frac{R^4}{4}$$

$$\therefore \frac{Ze.}{4\pi d R^3} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \qquad \therefore d = \frac{3Ze}{\pi R^3}$$

3. (c) If the volume charge density is constant then $E \propto r$.

4. (c) After colliding the top plate, the ball will gain negative charge and get repelled by the top plate and bounce back to the bottom plate.

5. (d) $I_{av} \propto \frac{Q}{t}$ ...(i)

Here $Q \propto V_0$ ...(ii)

Also $S = ut + \frac{1}{2}at^2$

$$h = \frac{1}{2}\frac{QE}{m}t^2 = \frac{1}{2}\left(\frac{Q \times 2V_0}{mh}\right) \times t^2$$

$\therefore$ $t \propto \frac{1}{V_0}$ – (iii) $[\because Q \propto V_0]$

From (i), (ii) and (iii)

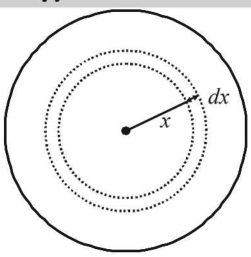
$$I_{av} \propto \frac{V_0}{1/V_0} = I_{av} \propto V_0^2$$

H. Assertion & Reason Type Questions

1. (a) Both the statements are true and statement-2 is the correct explanation of statement-1

I. Integer Value Correct Type

1. 2 Let us consider a spherical shell of radius x and thickness dx. The volume of this shell is $4\pi x^2(dx)$. The charge enclosed in this spherical shell is



$$dq = (4\pi x^2)dx \times kx^a$$

$\therefore$ $dq = 4\pi k x^{2+a} dx$.

For $r = R$:
The total charge enclosed in the sphere of radius R is

$$Q = \int_0^R 4\pi k\, x^{2+a}\, dx = 4\pi k \frac{R^{3+a}}{3+a}.$$

$\therefore$ The electric field at $r = R$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k R^{3+a}}{(3+a)R^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$$

For $r = R/2$:
The total charge enclosed in the sphere of radius $R/2$ is

$$Q' = \int_0^{R/2} 4\pi k\, x^{2+a} dx = \frac{4\pi k (R/2)^{3+a}}{3+a}$$

$\therefore$ The electric field at $r = R/2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\frac{(R/2)^{3+a}}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}\left(\frac{R}{2}\right)^{1+a}$$

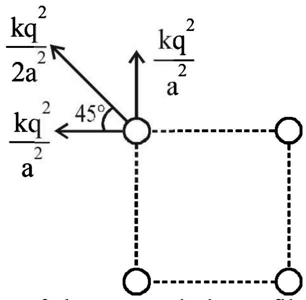
Given, $E_2 = \frac{1}{8}E_1$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{(3+a)}\left(\frac{R}{2}\right)^{1+a} = \frac{1}{2^3} \times \frac{1}{4\pi\varepsilon_0}\frac{4\pi k}{3+a}R^{1+a}$$

$\Rightarrow$ $1 + a = 3 \Rightarrow a = 2$

2. 3

$$F_{electric} = \frac{kq^2}{2a^2} + 2\left[\frac{kq^2}{a^2} \times \frac{1}{\sqrt{2}}\right] = \frac{q^2}{a^2} \times \text{constant}$$



As the system of charges and planar film is in equlibrium, therefore

$$\frac{q^2}{a^2} \times \text{constant} = \gamma a \times \text{constant}$$

$$\therefore \quad a = k\left(\frac{q^2}{\gamma}\right)^{1/3} \qquad \therefore \quad N = 3$$

3. 6 We suppose that the cavity is filled up by a positive as well as negative volume charge of ρ. So the electric field now produced at P is the superposition of two electric fields.

(a) The electric field created due to the infinitely long solid cylinder is

$$E_1 = \frac{\rho R}{4\varepsilon_0} \text{ directed towards the } +Y \text{ direction}$$

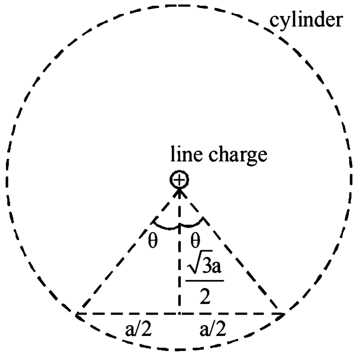
(b) The electric field created due to the spherical negative charge density

$$E_2 = \frac{\rho R}{96\varepsilon_0} \text{ directed towards the } -Y \text{ direction.}$$

$\therefore$ The net electric field is

$$E = E_1 - E_2 = \frac{1}{6}\left[\frac{23\rho R}{16\varepsilon_0}\right]$$

4. (6)



$$\tan\theta = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}}$$

$\therefore$ $\theta = 30°$

The flux through the dotted cylinder by Gauss's law is

$$\phi \text{ cylinder} = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$\therefore$ For 360° angle the flux is $\frac{\lambda L}{\varepsilon_0}$

$\therefore$ For 60° angle the flux will be $\frac{\lambda L}{6\varepsilon_0}$

Therefore n = 6

GP_3481

Section-B JEE Main/ AIEEE

1. (a) We know that $\frac{W_{AB}}{q} = V_B - V_A$

$\therefore V_B - V_A = \frac{2J}{20C} = 0.1\,\text{J/C} = 0.1\text{V}$

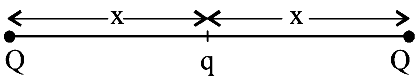
2. (b) The equivalent capacitance of n identical capacitors of capacitance C is equal to nC. Energy stored in this capacitor

$$E = \frac{1}{2}(nC)V^2 = \frac{1}{2}nCV^2$$

3. (b) Both the charges are identical and placed symmetrically about $ABCD$. The flux crossing $ABCD$ due to each charge is $\frac{1}{6}\left[\frac{q}{\epsilon_0}\right]$ but in opposite directions. Therefore the resultant is zero.

4. (d) For equilibrium of charge Q

$$K\frac{Q \times Q}{(2x)^2} + K\frac{Qq}{x^2} = 0 \Rightarrow q = -\frac{Q}{4}$$



5. (a) For an isolated sphere, the capacitance is given by

$$C = 4\pi \epsilon_0 r = \frac{1}{9 \times 10^9} \times 1 = 1.1 \times 10^{-10} F$$

6. (a) The flux entering an enclosed surface is taken as negative and the flux leaving the surface is taken as positive, by convention. Therefore the net flux leaving the enclosed surface $= \phi_2 - \phi_1$

$\therefore$ the charge enclosed in the surface by Gauss's law is

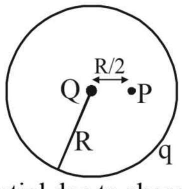
$q = \epsilon_0 (\phi_2 - \phi_1)$

7. (b) The capacitance of a parallel plate capacitor in which a metal plate of thickness t is inserted is given by

$C = \frac{\varepsilon_o A}{d - t}$. Here $t \to 0$ $\therefore C = \frac{\varepsilon_o A}{d}$

8. (c) Electric potential due to charge Q placed at the centre of the spherical shell at point P is

$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R/2} = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R}$$



Electric potential due to charge q on the surface of the spherical shell at any point inside the shell is

$$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

$\therefore$ The net electric potential at point P is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R} + \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

9. (d) The work done is stored as the potential energy. The potential energy stored in a capacitor is given by

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} \times \frac{\left(8 \times 10^{-18}\right)^2}{100 \times 10^{-6}} = 32 \times 10^{-32}\,\text{J}$$

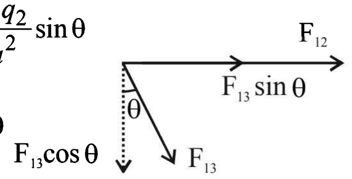
10. (b) Force on charge q_1 due to q_2 is $F_{12} = k\frac{q_1 q_2}{b^2}$

Force on charge q_1 due to q_3 is $F_{13} = k\frac{q_1 q_3}{a^2}$

The X - component of the force (F_x) on q_1 *is* $F_{12} + F_{13}\sin\theta$

$$\therefore F_x = k\frac{q_1 q_2}{b^2} + k\frac{q_1 q_2}{a^2}\sin\theta$$

$$\therefore F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$$



11. (d) $R_f = n^2 R_i$

Here n = 2 (length becomes twice)

$\therefore R_f = 4R_i$

New reresistance = 400 of R_i

$\therefore$ Increase = 300%

12. (d) $F \propto \frac{Q_A Q_C}{x^2}$

x is distance between the spheres. After first operation charge on B is halved i.e $\frac{Q}{2}$ and charge on third sphere becomes $\frac{Q}{2}$. Now it is touched to C, charge then equally distributes them selves to make potential same, hence charge on C becomes $\left(Q + \frac{Q}{2}\right)\frac{1}{2} = \frac{3Q}{4}$.

$$\therefore F_{new} \propto \frac{Q_C' Q_B'}{x^2} = \frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{2}\right)}{x^2} = \frac{3}{8}\frac{Q^2}{x^2}$$

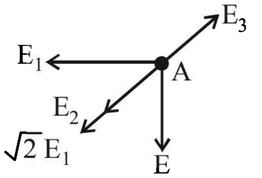
or $F_{new} = \frac{3}{8}F$

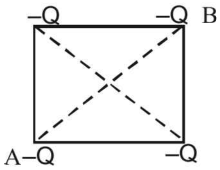
13. (d) $\frac{1}{2}mv^2 = \frac{kQq}{r} \Rightarrow \frac{1}{2}m(2v)^2 = \frac{kqQ}{r'} \Rightarrow r' = \frac{r}{4}$

14. (b) Net field at A should be zero

$\sqrt{2}\,E_1 + E_2 = E_3$

$$\therefore \frac{kQ\times\sqrt{2}}{a^2}+\frac{kQ}{(\sqrt{2}\,a)^2}=\frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2}$$





$$\Rightarrow \frac{Q\sqrt{2}}{1}+\frac{Q}{2}=2q \Rightarrow q=\frac{Q}{4}(2\sqrt{2}+1)$$

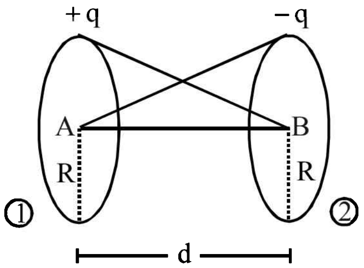
15. (c) At equilibrium, electric force on drop balances weight of drop.

$$qE=mg \Rightarrow q=\frac{mg}{E}=\frac{9.9\times10^{-15}\times10}{3\times10^4}=3.3\times10^{-18}\,\text{C}$$

16. (b) $\frac{-K2q}{(x-L)^2}+\frac{K8q}{x^2}=0 \Rightarrow \frac{1}{(x-L)^2}=\frac{4}{x^2}$

or $\frac{1}{x-L}=\frac{2}{x} \Rightarrow x=2x-2L$ or $x=2L$

17. (a)



$V_A=V_{\text{self}}+V_{\text{due to (2)}}$

$$\Rightarrow V_A=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

$V_B=V_{\text{self}}+V_{\text{due to (1)}}$

$$\Rightarrow V_B=\frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{R}+\frac{q}{\sqrt{R^2+d^2}}\right]$$

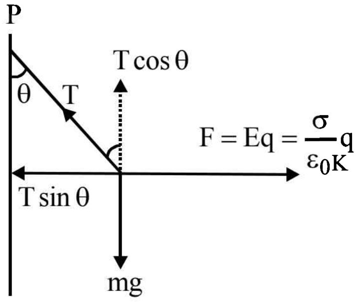
$\Delta V=V_A-V_B$

$$=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R}+\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

$$=\frac{1}{2\pi\varepsilon_0}\left[\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

18. (b) As n plates are joined, it means $(n-1)$ capacitor joined in parallel.
$\therefore$ resultant capacitance $=(n-1)\,C$

19. (c)



$$T\sin\theta=\frac{\sigma}{\varepsilon_0 K}.q \qquad \text{.... (i)}$$

$$T\cos\theta=mg \qquad \text{.... (ii)}$$

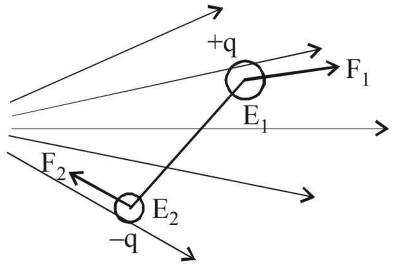
Dividing (i) by (ii),

$$\tan\theta=\frac{\sigma q}{\varepsilon_0 K.mg} \quad \therefore \sigma \propto \tan\theta$$

20. (c) Applying conservation of energy,

$$\frac{1}{2}CV^2=m.s\Delta T; \quad V=\sqrt{\frac{2m.s.\Delta T}{C}}$$

21. (c)



The electric field will be different at the location of the two charges. Therefore the two forces will be unequal. This will result in a force as well as torque.

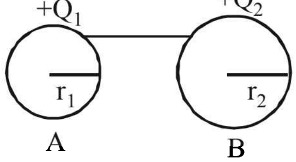
22. (a) $eV=\frac{1}{2}mv^2$

$$\Rightarrow v=\sqrt{\frac{2eV}{m}}=\sqrt{\frac{2\times1.6\times10^{-19}\times20}{9.1\times10^{-31}}}$$

$$=2.65\times10^6\ \text{m/s}$$

23. (c) After connection, $V_1=V_2$

$$\Rightarrow K\frac{Q_1}{r_1}=K\frac{Q_2}{r_2}$$



$$\Rightarrow \frac{Q_1}{r_1}=\frac{Q_2}{r_2}$$

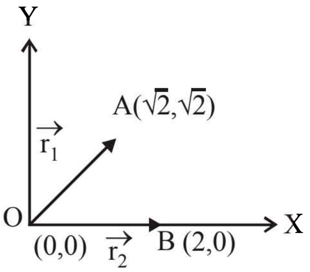
The ratio of electric fields

$$\frac{E_1}{E_2}=\frac{K\frac{Q_1}{r_1^2}}{K\frac{Q_2}{r_2^2}}=\frac{Q_1}{r_1^2}\times\frac{r_2^2}{Q_2}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{r_1 \times r_2^2}{r_1^2 \times r_2} \Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{2}{1}$$

Since the distance between the spheres is large as compared to their diameters, the induced effects may be ignored.

24. (c)



The distance of point $A(\sqrt{2}, \sqrt{2})$ from the origin,

$$OA = |\vec{r_1}| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = 2 \text{ units.}$$

The distance of point $B(2, 0)$ from the origin,

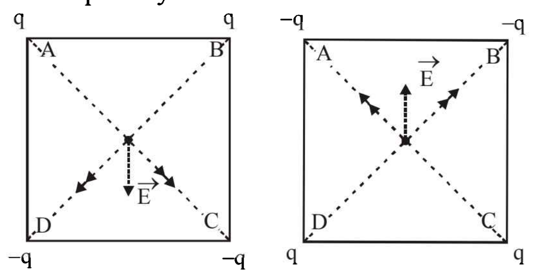
$$OB = |\vec{r_2}| = \sqrt{(2)^2 + (0)^2} = 2 \text{ units.}$$

Now, potential at A, $V_A = \frac{1}{4\pi \in_0} \cdot \frac{Q}{(OA)}$

Potential at B, $V_B = \frac{1}{4\pi \in_0} \cdot \frac{Q}{(OB)}$

$\therefore$ Potential difference between the points A and B is zero.

25. (a) As shown in the figure, the resultant electric fields before and after interchanging the charges will have the same magnitude, but opposite directions.
Also, the potential will be same in both cases as it is a scalar quantity.



26. (a) Here, $V(x) = \frac{20}{x^2 - 4}$ volt

We know that $E = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{20}{x^2 - 4}\right)$

or, $E = +\frac{40x}{(x^2 - 4)^2}$

At $x = 4\ \mu\text{m}$,

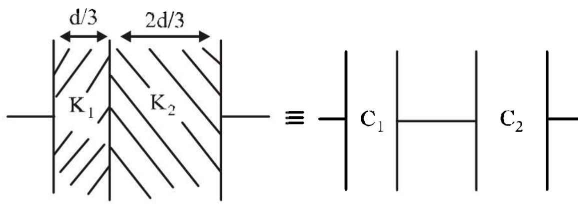
$$E = +\frac{40 \times 4}{(4^2 - 4)^2} = +\frac{160}{144} = +\frac{10}{9} \text{ volt}/\mu\text{m}.$$

Positive sign indicates that $\vec{E}$ is in +ve x-direction.

27. (a) The potential energy of a charged capacitor before removing the dielectric slat is $U = \frac{Q^2}{2C}$.
The potential energy of the capacitor when the dielectric slat is first removed and the reinserted in the gap between the plates is $U = \frac{Q^2}{2C}$
There is no change in potential energy, therefore work done is zero.

28. (b) Electronic charge does not depend on acceleration due to gravity as it is a universal constant.
So, electronic charge on earth
= electronic charge on moon
$\therefore$ Required ratio = 1.

29. (c)



The given capacitance is equal to two capacitances connected in series where

$$C_1 = \frac{k_1 \in_0 A}{d/3} = \frac{3k_1 \in_0 A}{d} = \frac{3 \times 3 \in_0 A}{d} = \frac{9 \in_0 A}{d}$$

and

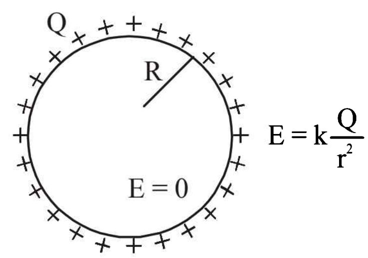
$$C_2 = \frac{k_2 \in_0 A}{2d/3} = \frac{3k_2 \in_0 A}{2d} = \frac{3 \times 6 \in_0 A}{2d} = \frac{9 \in_0 A}{d}$$

The equivalent capacitance C_{eq} is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{9 \in_0 A} + \frac{d}{9 \in_0 A} = \frac{2d}{9 \in_0 A}$$

$$\therefore C_{eq} = \frac{9}{2} \frac{\in_0 A}{d} = \frac{9}{2} \times 9\ pF = 40.5\,pF$$

30. (a) The electric field inside a thin spherical shell of radius R has charge Q spread uniformly over its surface is zero.



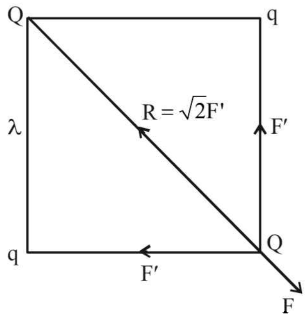
Outside the shell the electric field is $E = k\frac{Q}{r^2}$. These characteristics are represented by graph (a).

31. (c) $\frac{W_{PQ}}{q} = (V_Q - V_P)$

$\Rightarrow W_{PQ} = q(V_Q - V_P)$

$= (-100 \times 1.6 \times 10^{-19})(-4-10)$
$= +2.24 \times 10^{-16} J$

32. (d) Let F be the force between Q and Q. The force between q and Q should be attractive for net force on Q to be zero. Let F' be the force between Q and q. For equilibrium



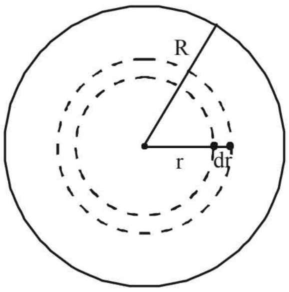
$\sqrt{2}F' = -F$

$\sqrt{2} \times k\frac{Qq}{\ell^2} = -k\frac{Q^2}{(\sqrt{2}\ell)^2}$

$\Rightarrow \frac{Q}{q} = -2\sqrt{2}$

33. (a) Statement 1 is true.
Statement 2 is true and is the correct explanation of (1)

34. (b)



Let us consider a spherical shell of thickness dx and radius x. The volume of this spherical shell $= 4\pi r^2 dr$.
The charge enclosed within shell

$= \frac{Qr}{\pi R^4}[4\pi r^2 dr]$

The charge enclosed in a sphere of radius r_1 is

$= \frac{4Q}{R^4}\int_0^{r_1} r^3 dr = \frac{4Q}{R^4}\left[\frac{r^4}{4}\right]_0^{r_1} = \frac{Q}{R^4}r_1^4$

$\therefore$ The electric field at point p inside the sphere at a distance r_1 from the centre of the sphere is

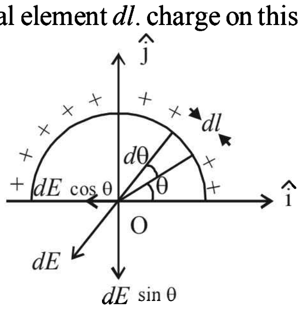
$$E = \frac{1}{4\pi \in_0}\frac{\left[\frac{Q}{R^4}r_1^4\right]}{r_1^2} = \frac{1}{4\pi \in_0}\frac{Q}{R^4}r_1^2$$

35. (c) Let us consider a differential element dl. charge on this element.

$dq = \left(\frac{q}{\pi r}\right)dl$

$= \frac{q}{\pi r}(r d\theta) \;(\because dl = r d\theta)$

$= \left(\frac{q}{\pi}\right)d\theta$



Electric field at O due to dq is

$$dE = \frac{1}{4\pi \in_0}.\frac{dq}{r^2} = \frac{1}{4\pi \in_0}.\frac{q}{\pi r^2}d\theta$$

The component $dE\cos\theta$ will be counter balanced by another element on left portion. Hence resultant field at O is the resultant of the component $dE\sin\theta$ only.

$\therefore E = \int dE\sin\theta = \int_0^{\pi}\frac{q}{4\pi^2 r^2 \in_0}\sin\theta d\theta$

$= \frac{q}{4\pi^2 r^2 \in_0}[-\cos\theta]_0^{\pi} = \frac{q}{4\pi^2 r^2 \in_0}(+1+1)$

$= \frac{q}{2\pi^2 r^2 \in_0}$

The direction of E is towards negative y-axis.

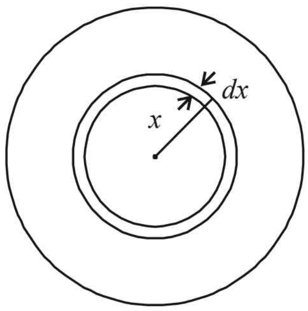
$\therefore \vec{E} = -\frac{q}{2\pi^2 r^2 \in_0}\hat{j}$

36. (a) Let us consider a spherical shell of radius x and thickness dx.
Charge on this shell

$dq = \rho.4\pi x^2 dx = \rho_0\left(\frac{5}{4} - \frac{x}{R}\right).4\pi x^2 dx$

$\therefore$ Total charge in the spherical region from centre to r $(r < R)$ is

$q = \int dq = 4\pi\rho_0\int_0^r\left(\frac{5}{4} - \frac{x}{R}\right)x^2 dx$



$= 4\pi\rho_0\left[\frac{5}{4}.\frac{r^3}{3} - \frac{1}{R}.\frac{r^4}{4}\right] = \pi\rho_0 r^3\left(\frac{5}{3} - \frac{r}{R}\right)$

$\therefore$ Electric field at r, $E = \frac{1}{4\pi \in_0}.\frac{q}{r^2}$

$= \frac{1}{4\pi \in_0}.\frac{\pi\rho_0 r^3}{r^2}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{\rho_0 r}{4\in_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

37. (d) At any instant

$T\cos\theta = mg$ ....(i)

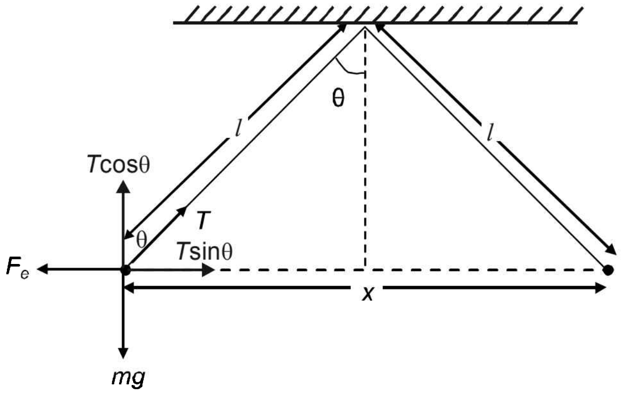
$T\sin\theta = F_e$ ....(ii)

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{F_e}{mg} \Rightarrow F_e = mg\tan\theta$

$\Rightarrow \quad \dfrac{kq^2}{x^2} = mg\tan\theta \Rightarrow q^2 \propto x^2 \tan\theta$

$\sin\theta = \dfrac{x}{2l}$

For small θ, $\sin\theta \approx \tan\theta$ $\quad \therefore\ q^2 \propto x^3$



$\Rightarrow \quad q\dfrac{dq}{dt} \propto x^2 \dfrac{dx}{dt}$

$\therefore \quad \dfrac{dq}{dt} = \text{const.}$

$\therefore \quad q \propto x^2.v \Rightarrow x^{3/2} \alpha x^2.v \quad [\because q^2 \propto x^3]$

$\Rightarrow \quad v \propto x^{-1/2}$

38. **(c)** Electric field

$E = -\dfrac{d\phi}{dr} = -2ar$ ....(i)

By Gauss's theorem

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$ ....(ii)

From (i) and (ii),

$q = -8\pi\varepsilon_0 ar^3$

$\Rightarrow \quad dq = -24\pi\varepsilon_0 ar^2\,dr$

Charge density, $\rho = \dfrac{dq}{4\pi r^2 dr} = -6\varepsilon_0 a$

39. **(c)** $E_{in} \propto r$

$E_{out} \propto \dfrac{1}{r^2}$

40. **(c)** The electric field inside a uniformly charged sphere is

$\dfrac{\rho . r}{3 \in_0}$

The electric potential inside a uniformly charged sphere

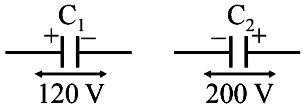
$= \dfrac{\rho R^2}{6 \in_0}\left[3 - \dfrac{r^2}{R^2}\right]$

∴ Potential difference between centre and surface

$= \dfrac{\rho R^2}{6 \in_0}[3-2] = \dfrac{\rho R^2}{6 \in_0}$

$\Delta U = \dfrac{q\rho R^2}{6 \in_0}$

41. **(b)**

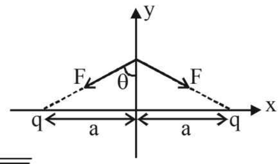


For potential to be made zero, after connection

$120\,C_1 = 200\,C_2 \quad \left[\because C = \dfrac{q}{v}\right]$

$\Rightarrow \quad 3C_1 = 5C_2$

42. **(a)** $\Rightarrow F_{net} = 2F\cos\theta$

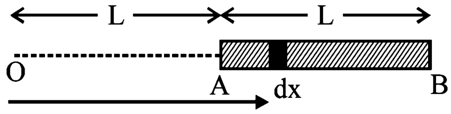


$F_{net} = \dfrac{2kq\left(\dfrac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2} \cdot \dfrac{y}{\sqrt{y^2+a^2}}$

$F_{net} = \dfrac{2kq\left(\dfrac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \Rightarrow \dfrac{kq^2 y}{a^3}$

So, $F \propto y$

43. **(d)**



Electric potential is given by,

$V = \int_L^{2L} \dfrac{kdq}{x} = \int_L^{2L} \dfrac{1}{4\pi\varepsilon_0} \dfrac{\left(\dfrac{q}{L}\right)dx}{x} = \dfrac{q}{4\pi\varepsilon_0 L}\ln(2)$

44. **(c)** Potential difference between any two points in an electric field is given by,

$dV = -\vec{E}.\vec{dx}$

$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx$

$V_A - V_O = -[10x^3]_0^2 = -80 J/C$

45. **(a)** Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by,

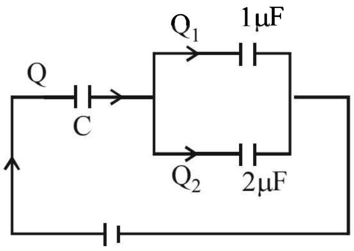
$E = \dfrac{\sigma}{K\varepsilon_0}$

Then, charge density

$\sigma = K\varepsilon_0 E$

$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4 \approx 6 \times 10^{-7}\ C/m^2$

46. **(d)**

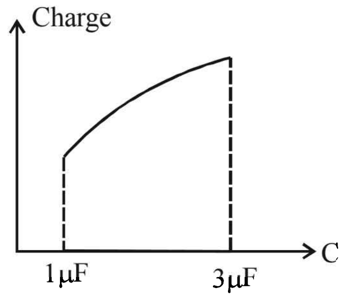


From figure, $Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q$

$$Q = E\left(\frac{C\times3}{C+3}\right)$$

$$\therefore Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$$

Therefore graph d correctly dipicts.



47. (a,b) We know, $V_0 = \frac{Kq}{R} = \text{Vsurface}$

Now, $V_i = \frac{Kq}{2R^3}(3R^2 - r^2)$ [For r < R]

At the centre of sphere r = 0. Here

$$V = \frac{3}{2}V_0$$

Now, $\frac{5}{4}\frac{Kq}{R} = \frac{Kq}{2R^3}(3R^2 - r^2)$

$$R_2 = \frac{R}{\sqrt{2}}$$

$$\frac{3}{4}\frac{Kq}{R} = \frac{Kq}{R^3}$$

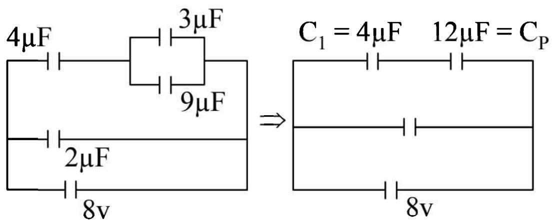
$$\frac{1}{4}\frac{Kq}{R} = \frac{Kq}{R_4}$$

$R_4 = 4R$

Also, $R_1 = 0$ and $R_2 < (R_4 - R_3)$

48. (c) Field lines originate perpendicular from positive charge and terminate perpendicular at negative charge. Further this system can be treated as an electric dipole.

49. (a)



Charge on C_1 is $q_1 = \left[\left(\frac{12}{4+12}\right)\times 8\right]\times 4 = 24\mu c$

The voltage across C_P is $V_P = \frac{4}{4+12}\times 8 = 2v$

∴ Voltage across 9μF is also 2V

∴ Charge on 9μF capacitor = 9 × 2 = 18μC

∴ Total charge on 4 μF and 9μF = 42μc

$$\therefore E = \frac{KQ}{r^2} = 9\times10^9\times\frac{42\times10^{-6}}{30\times30} = 420\ Nc^{-1}$$

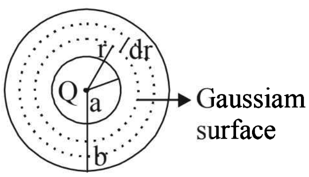
50. (c) Applying Gauss's law

$$\oint_S \vec{E}.\vec{ds} = \frac{Q}{\epsilon_0}$$

$$\therefore E\times 4\pi r^2 = \frac{Q + 4\pi a r^2 - 4\pi A a^2}{\epsilon_0}$$

$$\rho = \frac{dr}{dv}$$

$$Q = \rho 4\pi r^2$$



$$Q = \int_a^r \frac{A}{r}4\pi r^2 dr = 4\pi A[r^2 - a^2]$$

$$E = \frac{1}{4\pi\epsilon_0}\left[\frac{Q - 4\pi A a^2}{r^2} + 4\pi A\right]$$

For E to be independent of 'r'

$Q - 2\pi A a^2 = 0$

$$\therefore A = \frac{Q}{2\pi a^2}$$