PHYSICS AND MEASUREMENT

Chapter 1

REVIEW OF BASIC CONCEPTS

1. The SI System of Units

The internationally accepted standard units of the fundamental physical quantities are given in Table 1.1.

Table 1.1 *Fundamental SI Units*

Physical Quantity	*Name of the Unit*	*Symbol*
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Angle in a plane	radian	rad
Solid angle	steradian	sr

2. Dimensions of Physical Quantities

The dimensions of a physical quantity are the powers to which the fundamental units of mass (M), length (L) and time (T) must be raised to represent the unit of that quantity. The dimensional formula of a physical quantity is an expression that tells us how and which of the fundamental quantities enter into the unit of that quantity.

In *mechanics*, the dimensional formula is written in terms of the dimensions of mass, length and time (M, L and T). In *heat* and *thermodynamics*, in addition to M, L and T, we need to mention the dimension of temperature in kelvin (K). In *electricity and magnetism*, in addition to M, L and T, we need to mention the dimension of current or charge per unit time (A or QT^{-1}).

EXAMPLE 1 Find the dimensional formula of (a) velocity, (b) acceleration, (c) force, (d) work, (e) energy and (f) pressure.

SOLUTION

(a) Velocity $(v) = \dfrac{\text{distance}}{\text{time}} = \dfrac{dx}{dt} = \dfrac{[\text{L}]}{[\text{T}]}$

$= [\text{L T}^{-1}] = [\text{M}^0 \text{LT}^{-1}]$

(b) Acceleration $(a) = \dfrac{dv}{dt} = \dfrac{[\text{M}^0\text{ L T}^{-1}]}{[\text{T}]} = [\text{M}^0\text{ L T}^{-2}]$

(c) Force $(F) = ma = [\text{M}] \times [\text{M}^0\text{ L T}^{-2}] = [\text{M L T}^{-2}]$

(d) Work $(W) = Fx = [\text{M L T}^{-2}] \times [\text{L}] = [\text{M L}^2\text{ T}^{-2}]$

(e) Energy = work $= [\text{M L}^2\text{ T}^{-2}]$

(f) Pressure $= \dfrac{\text{force}}{\text{area}} = \dfrac{[\text{MLT}^{-2}]}{[\text{L}^2]} = [\text{ML}^{-1}\text{ T}^{-2}]$

Table 1.2 gives the dimensional formulae of some important physical quantities.

3. Principle of Homogeneity of Dimensions

Consider a simple equation,

$$A + B = C.$$

If this is an equation of Physics, i.e. if A, B and C are physical quantities, then this equation says that one physical quantity A, when added to another physical quantity B, gives a third physical quantity C. This equation will have no meaning in Physics if the nature (i.e. the dimensions) of the quantities on the left-hand side of the equation is not the same as the nature of the quantity on the right-hand side. For example, if A is a length, B must also be a length and the result of addition of A and B must express a length. In other words, the dimensions of both sides of a physical equation must be identical. This is called the *principle of homogeneity of dimensions*.

Table 1.2 *Dimensional Formulae of some Physical Quantities*

Physical Quantity	*Dimensional Formula*	*Physical Quantity*	*Dimensional Formula*
Area	$M^0L^2T^0$	Heat energy	ML^2T^{-2}
Volume	$M^0L^3T^0$	Entropy	$ML^2T^{-2}K^{-1}$
Density	$ML^{-3}T^0$	Specific heat	$M^0L^2T^{-2}K^{-1}$
Velocity	M^0LT^{-1}	Latent heat	$M^0L^2T^{-2}$
Acceleration	M^0LT^{-2}	Molar specific heat	$ML^2T^{-2}K^{-1}\ mol^{-1}$
Momentum	MLT^{-1}	Thermal conductivity	$MLT^{-3}K^{-1}$
Angular momentum	ML^2T^{-1}	Wien's constant	M^0LT^0K
Force	MLT^{-2}	Stefan's constant	$ML^0T^{-3}K^{-4}$
Energy, work	ML^2T^{-2}	Boltzmann's constant	$ML^2T^{-2}K^{-1}$
Power	ML^2T^{-3}	Molar gas constant	$ML^2T^{-2}K^{-1}\ mol^{-1}$
Torque, couple	ML^2T^{-2}	Electric charge	TA or Q
Impulse	MLT^{-1}	Electric current	A or QT^{-1}
Frequency	$M^0L^0T^{-1}$	Electric potential	$MLT^{-2}Q^{-1}$ or $ML^2T^{-3}A^{-1}$
Angular frequency	$M^0L^0T^{-1}$	Electric field	$MLT^{-2}Q^{-1}$ or $MLT^{-3}A^{-1}$
Angular acceleration	$M^0L^0T^{-2}$	Capacitance	$M^{-1}L^{-2}T^4A^2$ or $M^{-1}L^{-2}T^2Q^2$
Pressure	$ML^{-1}T^{-2}$	Inductance	$ML^2T^0Q^{-2}$ or $ML^2T^{-2}A^{-2}$
Elastic modulii	$ML^{-1}T^{-2}$	Resistance	$ML^2T^{-1}Q^{-2}$ or $ML^2T^{-3}A^{-2}$
Stress	$ML^{-1}T^{-2}$	Magnetic flux	$ML^2T^{-1}Q^{-1}$ or $ML^2T^{-2}A^{-1}$
Moment of inertia	ML^2T^0	Magnetic flux density or Magnetic induction field	$ML^0T^{-1}Q^{-1}$ or $ML^0T^{-2}A^{-1}$
Surface tension	ML^0T^{-2}	Permeability	MLQ^{-2} or $MLT^{-2}A^{-2}$
Viscosity	$ML^{-1}T^{-1}$	Permittivity	$M^{-1}L^{-3}T^2Q^2$ or $M^{-1}L^{-3}T^4A^2$
Gravitational constant	$M^{-1}L^3T^{-2}$	Planck's constant	ML^2T^{-1}

EXAMPLE 2 The distance x travelled by a body varies with time t as

$x = at + bt^2$, where a and b are constants.

Find the dimensions of a and b.

SOLUTION The dimensions of each term on the right hand of the given equation must be the same as those of the left hand side. Hence

Dimensions of at = dimensions of x

or $$[a] = \frac{[x]}{[t]} = \frac{[L]}{[T]} = [LT^{-1}] = [M^0LT^{-1}]$$

Dimensions of bt^2 = dimensions of x

or $$[b] = \frac{[x]}{[t^2]} = \frac{[L]}{[T^2]} = [LT^{-2}] = [M^0LT^{-2}]$$

EXAMPLE 3 The pressure P, volume V and temperature T of a gas are related as

$$\left(P + \frac{a}{V^2}\right)(V - b) = cT$$

where a, b, and c are constants. Find the dimensions of $\frac{a}{b}$.

SOLUTION Dimensions of $\frac{a}{V^2}$ = dimensions of P.

$\therefore$ Dimensions of a = dimensions of PV^2

Also dimensions of b = dimensions of V

$\therefore$ Dimensions of $\frac{a}{b} = \frac{[PV^2]}{[V]} = [PV] = [ML^{-1}T^{-2}] \times [L^3]$

$= [ML^2 T^{-2}]$

Note

1. Trigonometric functions (sin, cos, tan, cot, etc.) are dimensionless. The arguments of these functions are also dimensionless.
2. Exponential functions are dimensionless. Their exponents are also dimensionless.

EXAMPLE 4 When a plane wave travels in a medium, the displacement y of a particle located at x at time t is given by

$$y = a \sin(bt + cx)$$

where a, b and c are constant. Find the dimensions of $\frac{b}{c}$.

SOLUTION Terms bt and cx must be dimensionless. Hence

$$[b] = \frac{1}{[t]} = [T^{-1}]$$

and $$[c] = \frac{1}{[x]} = \frac{1}{[L]} = [L^{-1}]$$

$\therefore$ $$\left[\frac{b}{c}\right] = [LT^{-1}] = [M^0 LT^{-1}]$$

Note that the dimensions of a are the same as those of y.

EXAMPLE 5 In the expression

$$P = \frac{a^2}{b} e^{-ax}$$

P is pressure, x is a distance and a and b are constants. Find the dimensional formula for b.

SOLUTION $\left[\frac{a^2}{b}\right] = [P]$

Also ax is dimensionless. Hence $[a] = [L^{-1}]$.

$\therefore$ $$[b] = \frac{[a^2]}{[P]} = \frac{[L^{-2}]}{[ML^{-1}T^{-2}]} = [M^{-1}L^{-1}T^2]$$

The principle of homogeneity of dimensions can also be used to find the dependence of a physical quantity on other physical quantities.

EXAMPLE 6 In the expression

$$L = \frac{a + bx}{ct}$$

L is magnitude of angular momentum, x is a length, t is time and a, b, and c are constants. The dimensions of $\frac{a}{c}$ are the same as those of

(a) torque (b) moment of inertia
(c) work (d) Impulse

SOLUTION $L = \frac{a}{ct} + \frac{bx}{ct}$

$\therefore$ $$\left[\frac{a}{c}\right] = [Lt] = [mvr] \times [t]$$

$$= [M] \times [LT^{-1}] \times [L] \times [T]$$

$$= [ML^2 T^0]$$

The correct choice is (b).

EXAMPLE 7 Einstein's photoelectric equation is

$$K_{max} = h\nu - W_0$$

where K_{max} is the maximum kinetic energy of photoelectrons, ν is the frequency of incident radiation, W_0 is the work function of the metal and h is Planck's constant. Find the dimension formula of (a) h and (b) W_0.

SOLUTION The dimensions of $h\nu$ and W_0 are the same as those of K_{max}.

(a) $\therefore$ $[h\nu] = [K_{max}]$

$\Rightarrow$ $$[h] = \left[\frac{K_{max}}{\nu}\right] = \left[\frac{ML^2T^{-2}}{T^{-1}}\right] = [ML^2 T^{-1}]$$

(b) $[W_0] = [K_{max}] = [ML^2 T^{-2}]$

EXAMPLE 8 If pressure P is given by

$$P = \frac{a - \ell^2}{bt}$$

where ℓ is a length, t is time and a and b are constants, find the dimensional formula of b.

SOLUTION $[a] = [\ell^2] = [L^2]$

Also $[Pbt] = [a]$

$\Rightarrow$ $[ML^{-1}T^{-2}] \times [b] \times [T] = [L^2]$

$\Rightarrow$ $[ML^{-1}T^{-1}] \times [b] = [L^2]$

$\Rightarrow$ $$[b] = \frac{[L^2]}{[ML^{-1}T^{-1}]} = [M^{-1}L^3T]$$

EXAMPLE 9 Find the dimensional formula of (a) gravitation constant (G), (b) coefficient of viscosity (η), (c) specific heat capacity (s).

SOLUTION (a) From Newton's law of gravitation

$$F = \frac{G m_1 m_2}{r^2}$$

$\therefore$ $$[G] = \frac{[F] \times [r^2]}{[m_1] \times [m_2]} = \frac{[MLT^{-2}] \times [L^2]}{[M^2]}$$

$$= [M^{-1}L^3T^{-2}]$$

(b) The resistive force on a spherical body of radius r, moving with a velocity v in a fluid of coefficient of viscosity η is given by

$$F = 6\pi\eta r v$$

$$\therefore \quad [\eta] = \frac{[F]}{[r]\times[v]} = \frac{[\text{MLT}^{-2}]}{[\text{L}]\times[\text{LT}^{-1}]} = [\text{ML}^{-1}\,\text{T}^{-1}]$$

(c) $Q = ms\Delta T$
where Q is heat energy, m is the mass of the body and ΔT is the change in temperature.

$$\therefore \quad [s] = \frac{[Q]}{[m\Delta T]} = \frac{[\text{ML}^2\,\text{T}^{-2}]}{[\text{MK}]} = [\text{M}^0\,\text{L}^2\,\text{T}^{-2}\,\text{K}^{-1}]$$

EXAMPLE 10 In a standing wave, the displacement of a particle at x at time t is given by

$$y(x, t) = a \sin(bx)\cos(ct)$$

where a, b, and c are constants. Find the dimensions of $\frac{b}{c}$ are the same as those of

(a) wavelength (b) wave velocity

(c) $\frac{1}{\text{wavelength}}$ (d) $\frac{1}{\text{wave velocitiy}}$

SOLUTION Since bx and ct must be dimensionless, dimension of b is $[\text{L}^{-1}]$ and c is $[\text{T}^{-1}]$. Therefore

$$\left[\frac{b}{c}\right] = \frac{[\text{L}^{-1}]}{[\text{T}^{-1}]} = \frac{1}{[\text{LT}^{-1}]} = \frac{1}{\text{velocity}}$$

EXAMPLE 11 When a liquid of coefficient of viscosity η and density ρ flows through a pipe of radius r, the flow remains streamline if its velocity does not exceed a value v_c given by

$$v_c = \frac{k\eta}{\rho r}$$

where k is a constant called Reynold's constant. Show that k is dimensionless.

SOLUTION

$$k = \frac{v_c\,\rho\, r}{\eta}$$

$$\therefore \quad [k] = \frac{[v_c]\times[\rho]\times[r]}{[\eta]} = \frac{[\text{LT}^{-1}]\times[\text{ML}^{-3}]\times[\text{L}]}{[\text{ML}^{-1}\text{T}^{-1}]} = [\text{M}^0\,\text{L}^0\,\text{T}^0]$$

EXAMPLE 12 Find the dimensional formula for (a) electric field E and (b) magnetic field B in terms of M, L, T and A where A represents dimension of electric current.

SOLUTION

(a) $$E = \frac{F}{q} \Rightarrow [E] = \frac{[F]}{[q]}$$

$$\text{Current} = \frac{\text{charge}}{\text{time}}$$

or $$I = \frac{q}{t} \Rightarrow q = It \Rightarrow [q] = [\text{AT}]$$

$$\therefore \quad [E] = \frac{[\text{MLT}^{-2}]}{[\text{AT}]} = [\text{MLT}^{-3}\,\text{A}^{-1}]$$

(b) The force experienced by a charge q moving with a velocity $\boldsymbol{v}$ in a magnetic field $\mathbf{B}$ is given by

$$\mathbf{F} = q(\boldsymbol{v}\times\mathbf{B})$$

Magnitude of $\mathbf{F}$ is $F = qvB\sin\theta$. Since $\sin\theta$ is dimentionless

$$[B] = \frac{[F]}{[q]\times[v]} = \frac{[\text{MLT}^{-2}]}{[\text{AT}]\times[\text{LT}^{-1}]} = [\text{ML}^0\,\text{T}^{-2}\,\text{A}^{-1}]$$

EXAMPLE 13 The dimensions of $\frac{E}{B}$ are the same as those of

(a) current (b) $\frac{1}{\text{current}}$

(c) velocity (d) $\frac{1}{\text{velocity}}$

SOLUTION Lorentz force is given by

$$\mathbf{F} = \mathbf{q}[\mathbf{E} + \boldsymbol{v}\times\mathbf{B}]$$

Hence dimensions $\mathbf{E}$ are the same as those of $\boldsymbol{v}\times\mathbf{B}$.

$$\therefore \quad [E] = [v]\times[B]$$
($\because$ $\sin\theta$ is dimensionless)

$$\Rightarrow \quad \left[\frac{E}{B}\right] = [v], \text{ which is choice (c).}$$

EXAMPLE 14 Find the dimensional formula of (a) electrical permittivity (ε) and (b) magnetic permeability (μ).

SOLUTION The force between two charges q_1 and q_2 separated by a distance r in a medium of permittivity ε is

$$F = \frac{q_1\,q_2}{4\pi\,\varepsilon\, r^2}$$

$$\therefore \quad [\varepsilon] = \frac{[q_1]\times[q_2]}{[F]\times[r^2]}$$

$$= \frac{[\text{AT}]\times[\text{AT}]}{[\text{MLT}^{-2}]\times[\text{L}^2]}$$

$$= [\text{M}^{-1}\,\text{L}^{-3}\,\text{T}^4\,\text{A}^2]$$

(b) The magnetic field inside a long solenoid of length ℓ having N turns carrying a current I is given by

$$B = \frac{\mu N I}{\ell}$$

where μ is the permeability of the core. Therefore

$$[\mu] = \frac{[B]\times[\ell]}{[I]}$$

$$= \frac{[\text{ML}^0\,\text{T}^{-2}\,\text{A}^{-1}]\times[\text{L}]}{[\text{A}]}$$

$$= [\text{MLT}^{-2}\,\text{A}^{-2}]$$

EXAMPLE 15 The dimensions of $\mu\varepsilon$ are the same as those of

(a) current (b) $\frac{1}{(\text{current})^2}$

(c) velocity (d) $\frac{1}{(\text{velocity})^2}$

SOLUTION The velocity of an electromagnetic wave travelling in a medium of permeability μ and permittivity ε is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}}.$$

Hence the correct choice is (d).

EXAMPLE 16 Find the dimensions of $\varepsilon_0 E^2$.

SOLUTION The energy density u (energy per unit volume) of the electric field when an electromagnetic wave travels in a vacuum is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

$$\therefore \quad [\varepsilon_0 E^2] = [u] = \frac{[\text{energy}]}{[\text{volume}]}$$

$$= \frac{(\text{ML}^2\,\text{T}^{-2})}{[\text{L}^3]}$$

$$= [\text{ML}^{-1}\,\text{T}^{-2}]$$

EXAMPLE 17 Find the dimensional formula of (a) electrical resistance and (b) electrical conductivity.

SOLUTION

(a) $V = IR$. Therefore $[R] = \frac{[V]}{[I]}$ (1)

Now $V = \frac{W}{q}$.

Therefore $[V] = \frac{[W]}{[q]} = \frac{[\text{ML}^2\,\text{T}^{-2}]}{[\text{AT}]}$

$$= [\text{ML}^2\text{T}^{-3}\text{A}^{-1}] \qquad (2)$$

Using (2) in (1) we get

$$[R] = \frac{[\text{ML}^2\,\text{T}^{-3}\text{A}^{-1}]}{[\text{A}]}$$

$$= [\text{ML}^2\text{T}^{-3}\text{A}^{-2}]$$

(b) Electrical resistivity $\rho = \frac{RA}{\ell}$. Electrical conductivity is defined as

$$\sigma = \frac{1}{\rho} = \frac{\ell}{RA}; \quad A = \text{area}$$

$$\therefore \quad [\sigma] = \frac{[\ell]}{[R]\times[A]} = \frac{[\text{L}]}{[\text{ML}^2\,\text{T}^{-3}\,\text{A}^{-2}]\times[\text{L}^2]}$$

$$= [\text{M}^{-1}\,\text{L}^{-3}\,\text{T}^3\,\text{A}^2]$$

EXAMPLE 18 Find the dimensional formula of (a) capacitance C and (b) inductance L.

SOLUTION (a) $Q = CV$

$$\therefore \quad [C] = \frac{[Q]}{[V]} = \frac{[\text{AT}]}{[\text{ML}^2\,\text{T}^{-3}\text{A}^{-1}]}$$

$$= [\text{M}^{-1}\,\text{L}^{-2}\,\text{T}^4\,\text{A}^2]$$

(b) $\phi = LI$, ϕ = magnetic flux (**B**.**A**)

$$\therefore \quad [L] = [\phi]\times[I]^{-1} = [B]\times[A]\times[I]^{-1}$$

$$= [\text{ML}^0\,\text{T}^{-2}\text{A}^{-1}]\times[\text{L}^2]\times[\text{A}^{-1}]$$

$$= [\text{ML}^2\,\text{T}^{-2}\,\text{A}^{-2}]$$

EXAMPLE 19 If T denotes dimension of time period, the dimensions of CL are

(a) T^{-2} (b) T^{-1}

(c) T (c) T^2

SOLUTION The angular frequency of an LC circuit is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad LC = \frac{1}{\omega^2} = \frac{1}{4\pi^2\nu^2} = \frac{\text{T}^2}{4\pi^2}$$

So the correct choice is (d).

EXAMPLE 20 Find the dimensions of $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.

SOLUTION $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \sqrt{\frac{L}{R^2C}} = \sqrt{\frac{L/R}{RC}}$

$\frac{L}{R}$ is the time constant of an *LR* circuit and *RC* is the time constant of an *RC* circuit. Hence *Q* is dimensionless.

EXAMPLE 21 The time period (t) of a simple pendulum may depend upon m the mass of the bob, l the length of the string and g the acceleration due to gravity. Find the dependence of t on m, l and g.

SOLUTION

Let $t \propto m^a l^b g^c$

or $t = k\, m^a l^b g^c$,

where k is a dimensionless constant.

Writing the dimensions of each quantity, we have

$$[\text{T}] = [\text{M}]^a [\text{L}]^b [\text{LT}^{-2}]^c$$

or $[\text{M}^0 \text{L}^0 \text{T}] = [\text{M}^a \text{L}^{b+c} \text{T}^{-2c}]$

According to the principle of homogeneity of dimensions, the dimensions of all the terms on either side of this equation must be the same. Equating the powers of M, L and T, we have

$a = 0$, $b + c = 0$ and $-2c = 1$,

which give $b = \frac{1}{2}$ and $c = -\frac{1}{2}$.

Hence $t = k\, m^0\, l^{1/2}\, g^{(-1/2)}$

$\Rightarrow$ $t = k\sqrt{\frac{l}{g}}$

Thus t is independent of the mass of the bob and is directly proportional to $\sqrt{l}$ and inversely proportional to $\sqrt{g}$.

EXAMPLE 22 Assuming that the time period T of a planet depends upon its mean distance r from the sun, gravitation constant G and mass m of the sun, show, by using dimensional considerations that $T \propto r^{3/2}$.

SOLUTION Let $T \propto r^a G^b m^c$

or $T = k\, r^a G^b m^c$

Substituting dimensions,

$$[\text{M}^0 \text{L}^0 \text{T}] = [\text{L}]^a \times [\text{M}^{-1} \text{L}^3 \text{T}^{-2}]^b \times [\text{M}]^c$$
$$= [\text{M}^{-b+c}] \times [\text{L}^{a+3b}] \times [\text{T}^{-2b}]$$

Equating powers of M, L and T, we get

$-b + c = 0$, $a + 3b = 0$, and $-2b = 1$.

These equations give $a = \frac{3}{2}$, $b = -\frac{1}{2}$ and $c = -\frac{1}{2}$. Hence

$$T = k\, r^{3/2}\, G^{-1/2}\, m^{-1/2}$$

Thus $T \propto r^{3/2}$. This is Kepler's law of periods.

EXAMPLE 23 The volume of a liquid flowing per second (Q) through a uniform pipe depends upon r the radius of the pipe, η the coefficient of viscosity of the liquid and pressure gradient $\left(\frac{p}{\ell}\right)$ between the ends of the pipe. Dimensional considerations show that Q is proportional to

(a) r (b) r^2
(c) r^3 (d) r^4

SOLUTION $Q = k\left(\frac{p}{\ell}\right)^a r^b \eta^c$

$$\therefore \quad [\text{L}^3 \text{T}^{-1}] = \frac{[\text{ML}^{-1}\text{T}^{-2}]^a}{[\text{L}]^a} \times [\text{L}^b] \times [\text{ML}^{-1}\text{T}^{-1}]^c$$
$$= [\text{M}^{a+c}] \times [\text{L}^{-2a+b-c}] \times [\text{T}^{-2a-c}]$$

$\therefore$ $a + c = 0$, $-2a + b - c = 3$ and $-2a - c = 1$. These equations give $a = 1$, $b = 4$ and $c = -1$. Thus

$$Q = k\,\frac{pr^4}{\ell\eta}$$

$\therefore$ $Q \propto r^4$, which is choice (d).

EXAMPLE 24 The speed (v) of transverse waves travelling on a string depends on tension (T) with which the string is stretched and mass per unit length (μ) of the string. Using dimensional considerations, obtain the expression for v in terms of T and μ.

SOLUTION Let $v = k\, T^a \mu^b$

$$\therefore \quad [\text{M}^0 \text{LT}^{-1}] = [\text{MLT}^{-2}]^a \times \left[\frac{\text{M}}{\text{L}}\right]^b$$
$$= [\text{M}^{(a+b)}] \times [\text{L}^{(a-b)}] \times [\text{T}^{-2a}]$$

Equating powers of M, L and T,

$a + b = 0$, $a - b = 1$, $-2a = -1$. Solving these equations we get $a = +\frac{1}{2}$, $b = -\frac{1}{2}$. Hence

$$v = k\, T^{\frac{1}{2}}\, \mu^{-\frac{1}{2}} = k\sqrt{\frac{T}{\mu}}$$

where k is a dimensionless constant.

4. Significant Figures

The number significant figure in any measurement indicates the degree of precision of that measurement. The degree of precision is determined by the least count of the measuring instrument. Suppose a length measured by a metre scale (of least count = 0.1 cm) is 1.5 cm, then it has two significant figures, namely 1 and 5. Measured with a vernier callipers (of least count = 0.01 cm) the same length is 1.53 cm and it then has three significant figures. Measured with a screw gauge (of least count = 0.001 cm) the same length may be 1.536 cm which has four significant figures.

It must be clearly understood that we cannot increase the accuracy of a measurement of changing the unit. For example, suppose a measurement of mass yields a value 39.4 kg. It is understood that the measuring instrument has a least count of 0.1 kg. In this measurement, three figures 3, 9 and 4 are significant. If we change 39.4 kg to 39400 g or 39400000 mg, we cannot change the accuracy of measurement. Hence 39400 g or 39400000 mg still have three significant figures; the zeros only serve to indicate only the magnitude of measurement.

Estimation of Appropriate Significant Figures in Calculations

The importance of significant figures lies in calculation to find the result of addition or multiplication of measured quantities having a different number of significant figures. The least accurate quantity determines the accuracy of the sum or product. The result must be rounded off to the appropriate digit.

Rules for Rounding off

The following rules are used for dropping figures that are not significant

1. If the digit to be dropped is less than 5, the next (preceding) digit to be retained is left unchanged. For example, if a number 5.34 is to be rounded off to two significant figures, the digit to be dropped is 4 which is less than 5. Hence the next digit, namely 3, is not changed. The result of the indicated rounding-off is therefore, 5.3.
2. If the digit to be dropped is more than 5, the preceding digit to be retained is increased by 1. For examples 7.536 is rounded off as 7.54 to three significant figures.
3. If the digit to be dropped happens to be 5, then
   (a) the preceding digit to be retained is increased by 1 if it is odd, or
   (b) the preceding digit is retained unchanged if it is even.

   For example, 6.75 is rounded off to 6.8 to two significant figures and 4.95 is rounded off to 5.0 but 3.45 is rounded off to 3.4.

Rule for Finding Significant Figures

1. For addition and subtraction, we use the following rule.

   Find the sum or difference of the given measured quantities and then round off the final result such that it has the same number of digits after the decimal place as in the least accurate quantity (i.e., the quantity which has the least number of significant figures)

EXAMPLE 25 Four objects have masses 2.5 kg, 1.54 kg, 3.668 kg and 5.1278 kg. Find the total mass up to appropriate significant figures.

SOLUTION

$$M = 2.5 + 1.54 + 3.668 + 5.1278 = 12.8358 \text{ kg}$$

In this example, the least accurate quantity is 2.5 kg. This mass is accurate only up to the first decimal place in kg. Hence the final result much be rounded off to the first decimal place in kg. The correct result up to appropriate significant figures is $M = 12.8$ kg.

2. We use the following rule to determine the number of significant figures in the result of multiplication and division of various physical quantities.

   Do not worry about the number of digits after the decimal place. Round off the result so that it has the same number of significant figures as in the least accurate quantity.

EXAMPLE 26 A man runs 100.2 m in 10.3 s. Find his average speed up to appropriate significant figure.

SOLUTION

$$\text{Average speed } (v) = \frac{100.2\text{ m}}{10.3\text{s}} = 9.728155 \text{ ms}^{-1}$$

The distance 100.2 m has four significant figures but the time 10.3 s has only three. Hence the value of the result must be round off to three significant figures. The correct result is $v = 9.73 \text{ ms}^{-1}$

5. Least Counts of Some Measuring Instruments

1. Least count of metre scale = 1 mm = 0.1 cm
2. Vernier constant (or least count) of vernier callipers

   = value of 1 main scale division – value of 1 vernier scale division = 1 M.S.D. – 1 V.S.D

   Let the value of 1 M.S.D = a unit

   If n vernier scale divisions coincide with m main scale divisions, then value of

$$1.\text{V.S.D} = \frac{m}{n} \text{ of 1 M.S.D} = \frac{ma}{n} \text{ unit}$$

$$\therefore \quad \text{Least count} = a - \frac{ma}{n} = \left(1 - \frac{m}{n}\right) a \text{ unit}$$

3. Least count of a micrometer screw is found by the formula

$$\text{Least count} = \frac{\text{Pitch of screw}}{\text{Total number of divisions on circular scale}}$$

where pitch = lateral distance moved in one complete rotation of the screw.

EXAMPLE 27 In a vernier callipers, 19 divisions of the main scale exactly coincide with 20 divisions of the vernier scale. If the smallest division of the main scale is 1.0 mm, find the vernier constant of the callipers.

SOLUTION Value of 1 M.S.D. = 1.0 mm

20 divisions of the vernier scale = 19 M.S.D.

$$= 19 \times 1.0\,\text{mm} = 19\,\text{mm}$$

$$\therefore \quad \text{Value of 1 V.S.D.} = \frac{19}{20}\,\text{mm}$$

$$\text{Least count or vernier constant} = \text{1 M.S.D.} - \text{1 V.S.D}$$

$$= 1.0\,\text{mm} - \frac{19}{20}\,\text{mm}$$

$$= \frac{1}{20}\,\text{mm} = 0.05\,\text{mm}$$

$$= 0.005\,\text{cm}$$

EXAMPLE 28 A physical quantity $X = \dfrac{A^2 B}{c^{1/3}\sqrt{D}}$ is calculated by using measured quantities A, B, C and D.

If the errors in the measurement of A, B, C and D are 1%, 2%, 3% and 4% respectively, find the percentage error in the measurement of X.

SOLUTION Given $X = \dfrac{A^2 B}{c^{1/3}\sqrt{D}}$

$$\therefore \quad \frac{\Delta X}{X} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{1}{3}\frac{\Delta C}{C} + \frac{1}{2}\frac{\Delta D}{D}$$

$$= 2 \times 1\% + 2\% + \frac{1}{3} \times 3\% + \frac{1}{2} \times 4\%$$

$$= 2\% + 2\% + 1\% + 2\%$$

$$= 7\%$$

EXAMPLE 29 The length and breadth of a rectangular lamina are $L = (2.6 \pm 0.1)$ cm and $B = (1.8 \pm 0.2)$ cm. Find the area of the lamina up to appropriate significant figures stating the error limits.

SOLUTION Area is $A = 2.6 \times 1.8 = 4.68\text{ cm}^2$

Now $\quad A = L \times B$

$$\therefore \quad \frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta B}{B}$$

$$= \frac{0.1}{2.6} + \frac{0.2}{1.8}$$

$$= 0.038 + 0.111$$

$$= 0.149$$

$$\therefore \quad \Delta A = 0.149 \times A = 0.149 \times 4.68$$

$$= 0.697\text{ cm}^2$$

Since the error is in the first decimal place, we round off $\Delta A = 0.7\text{ cm}^2$. The value of A cannot be correct upto the second decimal place. So $A = 4.7\text{ cm}^2$. The result of the measurement is written as

$$A = (4.7 \pm 0.7)\,\text{cm}^2$$

EXAMPLE 30 The error in the measurement of diameter of a sphere is 1%. The percentage error in the measurement of the volume is

(a) 1% (b) 2%
(c) 3% (d) 4%

SOLUTION

$$V = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \frac{\pi}{6}D^3 \;\left(\because R = \frac{D}{2}\right)$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{3\Delta D}{D} = 3 \times 1\% = 3\%$$

EXAMPLE 31 Two resistors $R_1 = (2.2 \pm 0.1)\,\Omega$ and $R_2 = (8.0 \pm 0.2)\,\Omega$ are connected in series. The resistance R_s of the series combination is

(a) $(10.2 \pm 0.1)\,\Omega$ (b) $(10.2 \pm 0.2)\,\Omega$
(c) $(10.2 \pm 0.3)\,\Omega$ (d) $(10.2 \pm 0.4)\,\Omega$

SOLUTION $R_s = R_1 + R_2 = 2.2 + 8.0 = 10.2\,\Omega$

$$\Delta R_s = \Delta R_1 + \Delta R_2 = 0.1 + 0.2 = 0.3\,\Omega$$

$\therefore \quad R_s = (10.2 \pm 0.3)\,\Omega$. So the correct choice is (c).

EXAMPLE 32 In Ex. 31 above, if the resistors R_1 and R_2 are connected in parallel, the resistance R_p of the parallel combination is

(a) $(1.725 \pm 0.171)\,\Omega$ (b) $(1.73 \pm 0.17)\,\Omega$
(c) $(1.7 \pm 0.2)\,\Omega$ (d) $(1.725 \pm 0.2)\,\Omega$

SOLUTION

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_2}{R_s} = \frac{2.2 \times 8.0}{10.2} = 1.725\,\Omega$$

$$\therefore \quad \frac{\Delta R_p}{R_p} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_s}{R_s}$$

$$= \frac{0.1}{2.2} + \frac{0.2}{8.0} + \frac{0.3}{10.2}$$
$$= 0.045 + 0.025 + 0.029$$
$$= 0.099$$

$\therefore \quad \Delta R_p = 0.099 \times R_p = 0.099 \times 1.725 = 0.171 = 0.2\,\Omega$

$\therefore \quad R_p = (1.7 \pm 0.2)\,\Omega$

The correct choice is (c).

6. Order of Accuracy: Proportionate Error

The order of accuracy of the result of measurements is determined by the least counts of the measuring instruments used to make those measurements. Suppose a length x is measured with a metre scale, then the error in x is $\pm \Delta x$, where Δx = least count of metre scale = 0.1 cm. If the same length is measured with vernier callipers of least count 0.01 cm, then $\Delta x = 0.01$ cm.

Fractional or proportionate error is defined as $\dfrac{\Delta x}{x}$.

Maximum percentage error = $\dfrac{\Delta x}{x} \times 100$.

1. **Error in Sum:** Suppose a quantity is given by

   $a = x + y$

   Then $\Delta a = \Delta x + \Delta y$ is the maximum error and

   $$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x + y)}$$

2. **Error in Difference:** If $a = x - y$, then the maximum error is

   $\Delta a = \Delta x + \Delta y$

   We take the worst case in which errors add up.

   $$\frac{\Delta a}{a} = \frac{\Delta x + \Delta y}{(x - y)}$$

3. **Error in Product and Division:** Suppose we determine the value of a physical quantity u by measuring three quantities x, y and z whose true values are related to u by the equation

   $u = x^{\alpha} y^{\beta} z^{-\gamma}$

   Let the expected small errors in the measurement of quantities x, y and z be respectively $\pm \delta x$, $\pm \delta y$ and $\pm \delta z$ so that the error in u by using these observed quantities is $\pm \delta u$. The numerical values of δx, δy and δz are given by the least count of the instruments used to measure them.

   Taking logarithm of both sides we have

   $$\log u = \alpha \log x + \beta \log y - \gamma \log z$$

   Partial differentiation of the above equation gives

   $$\frac{\Delta u}{u} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} - \gamma \frac{\Delta z}{z}$$

   The proportional or relative error in u is $\Delta u/u$. The values of Δx, Δy and Δz may be positive or negative and in some cases the terms on the right hand side may counteract each other. This effect cannot be relied upon and it is necessary to consider the worst case which is the case when all errors *add* up giving an error Δu given by the equation:

   $$\left(\frac{\Delta u}{u}\right)_{\max} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} + \gamma \frac{\Delta z}{z}$$

   Thus to find the maximum proportional error in u, multiply the proportional errors in each factor (x, y and z) by the numerical value of the power to which each factor is raised and then add all the terms so obtained.

   The sum thus obtained will give the maximum proportional error in the result of u. When the proportional error of a quantity is multiplied by 100, we get the percentage error of that quantity.

EXAMPLE 33 In an experiment for determining density (ρ) of a rectangular metal block, a student makes the following measurements.

Mass of block (m) = 39.3 g
Length of block (x) = 5.12 cm
Breadth of block (y) = 2.56 cm
Thickness of block (z) = 0.37 cm

The uncertainty in the measurement of m is $\pm$ 0.1 g and in the measurement of x, y and z is $\pm$ 0.01 cm. Find the value of ρ (in g cm^{-3}) up to appropriate significant figures, stating the uncertainty in the value of ρ.

SOLUTION

$$\rho = \frac{m}{xyz} = \frac{39.3}{5.12 \times 2.56 \times 0.37} = 8.1037 \text{ g cm}^{-3}$$

$$\left(\frac{\Delta \rho}{\delta}\right)_{\max} = \frac{\Delta m}{m} + \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$= \frac{0.1}{39.3} + \frac{0.01}{5.12} + \frac{0.01}{2.56} + \frac{0.01}{0.37}$$

$$= 0.0353$$

$\therefore \quad \Delta \rho = 0.0353 \times \rho = 0.0353 \times 8.1037$

$\quad = 0.286 \text{ gcm}^{-3}$

Round off error $\Delta \rho$ to the first significant figure as $\Delta \rho = 0.3 \text{ gcm}^{-3}$.

Hence $\rho = 8.1037 \text{ gm}^{-3}$ is not accurate to the fourth decimal place. In fact, it is accurate only up to the first decimal place. Hence the value of ρ much be rounded off as 8.1 and the result of the measurement is written as

$$\rho = (8.1 \pm 0.3) \text{ g cm}^{-3}$$

An Important Tip

To find the dimensional formula of the required quantity, recall any relation which relates that quantity with other quantities whose dimensions we already know. For example, to find the dimensional formula for capacitance C, we can use relation

$$Q = CV \text{ or } C = \frac{\varepsilon A}{d} \text{ or } U = \frac{1}{2} CV^2.$$

Similarly, to find the dimensional formula for magnetic field B, we can use relation $F = q\,v\,B$ or $F = BIL$. or $B = \mu nI$ (here n = Number of turns per unit length).

EXAMPLE 34 Find the dimensional formula of Bohr magneton.

SOLUTION Bohr magneton $= \frac{eh}{4\pi m_e}$

$$= \frac{[\mathrm{AT}]\times[\mathrm{ML^2T^{-1}}]}{\mathrm{M}}$$

$$= [\mathrm{M^0L^2T^0A}]$$

The SI unit of Bohr magneton is ampere (metre)^2 (or Am^2)

Note

1. Errors always add; they never cancel other.
2. The quantity which is raised to the highest power contributes the maximum error and hence it must be measured to a high degree of accuracy.

Applications

1. For a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$
2. For a sphere of radius r,

$$\text{Surface area } A = 4\pi r^2 \Rightarrow \frac{\Delta A}{A} = \frac{2\Delta r}{r}$$

$$\text{Volume } V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta V}{V} = \frac{3\Delta r}{r}$$

3. Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\Rightarrow \frac{\Delta g}{g} = \frac{2\Delta R}{R} + \frac{\Delta M}{M}$$

4. For resistances connected in series

$$R_s = R_1 + R_2 \Rightarrow \frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

5. For resistances connected in parallel

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow -\frac{\Delta R_p}{R_p^2} = -\frac{\Delta R_1}{R_1^2} - \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \frac{\Delta R_p}{R_P^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

6. Kinetic energy K and linear momentum p are related as

$$K = \frac{p^2}{2m} \Rightarrow \frac{\Delta K}{K} = \frac{2\Delta p}{p}$$

SECTION 1 Multiple Choice Questions with One Correct Choice

Level A

1. Which one of the following is *not* a unit of length?
   (a) angström (b) light year
   (c) fermi (d) radian
2. The unit of impulse is the same as that of
   (a) moment of force
   (b) linear momentum
   (c) rate of change of linear momentum
   (d) force
3. Which pair of quantities has dimensions different from the other three pairs?
   (a) Impulse and linear momentum
   (b) Planck's constant and angular momentum
   (c) Moment of inertia and moment of force
   (d) Young's modulus and pressure
4. The dimensions of the coefficient of viscosity are
   (a) $\mathrm{ML^2T^{-2}}$ (b) $\mathrm{MLT^{-1}}$
   (c) $\mathrm{ML^{-1}T^{-1}}$ (d) $\mathrm{ML^{-1}T^{-2}}$
5. The dimensions of surface tension are
   (a) $\mathrm{ML^0T^{-2}}$ (b) $\mathrm{MLT^{-2}}$
   (c) $\mathrm{ML^{-1}T^{-2}}$ (d) $\mathrm{ML^{-2}T^{-2}}$
6. The SI unit of the universal gravitational constant G is
   (a) Nm kg^{-2} (b) Nm^2 kg^{-2}
   (c) Nm^2 kg^{-1} (d) Nm kg^{-1}

7. The dimensions of the coefficient of thermal conductivity are
(a) $MLT^{-3} K^{-1}$ (b) $MLT^{-2} K^{-1}$
(c) $MLT^{-1} K^{-1}$ (d) $MLT^{-2} K^{-2}$

8. The SI unit of Stefan's constant is
(a) $W s^{-1} m^{-2} K^{-4}$ (b) $J s m^{-2} K^{-4}$
(c) $J s^{-1} m^{-2} K^{-1}$ (d) $Wm^{-2} K^{-4}$

9. What is the physical quantity whose dimensions are ML^2T^{-2}?
(a) kinetic energy (b) pressure
(c) momentum (d) power

10. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(a) torque (b) surface tension
(c) viscosity (d) stress

11. The dimensions of angular momentum are
(a) MLT^{-1} (b) ML^2T^{-1}
(c) $ML^{-1}T$ (d) ML^0T^{-2}

12. The gravitational force F between two masses m_1 and m_2 separated by a distance r is given by $F = \dfrac{G m_1 m_2}{r^2}$ where G is the universal gravitational constant. What are the dimensions of G?
(a) $M^{-1}L^3T^{-2}$ (b) ML^3T^{-2}
(c) ML^2T^{-3} (d) $M^{-1}L^2T^{-3}$

13. Time period T of a simple pendulum may depend on m, the mass of the bob, l, the length of the string and g, the acceleration due to gravity, i.e.

$$T \propto m^a\, l^b\, g^c$$

What are the values of a, b and c?
(a) $0, \frac{1}{2}, -\frac{1}{2}$ (b) $0, -\frac{1}{2}, \frac{1}{2}$
(c) $\frac{1}{2}, 0, -\frac{1}{2},$ (d) $-\frac{1}{2}, 0, \frac{1}{2}$

14. The volume V of water passing any point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation

$$V \propto A^{\alpha}\, u^{\beta}\, t^{\gamma}$$

which one of the following will be true?
(a) $\alpha = \beta = \gamma$ (b) $\alpha \neq \beta = \gamma$
(c) $\alpha = \beta \neq \gamma$ (d) $\alpha \neq \beta \neq \gamma$

15. The frequency n of vibrations of uniform string of length l and stretched with a force F is given by

$$n = \frac{p}{2l}\sqrt{\frac{F}{m}}$$

where p is the number of segments of the vibrating string and m is a constant of the string. What are the dimensions of m?
(a) $M L^{-1} T^{-1}$ (b) $M L^{-3} T^0$
(c) $M L^{-2} T^0$ (d) $M L^{-1} T^0$

16. The dimensions of specific heat capacity are
(a) $MLT^{-2} K^{-1}$ (b) $ML^2 T^{-2} K^{-1}$
(c) $M^0L^2T^{-2} K^{-1}$ (d) $M^0LT^{-2} K^{-1}$

17. What are the dimensions of latent heat?
(a) $ML^2 T^{-2}$ (b) $ML^{-2} T^{-2}$
(c) $M^0 LT^{-2}$ (d) $M^0 L^2T^{-2}$

18. What are the dimensions of Boltzmann's constant?
(a) $MLT^{-2} K^{-1}$ (b) $ML^2 T^{-2} K^{-1}$
(c) $M^0LT^{-2} K^{-1}$ (d) $M^0L^2T^{-2} K^{-1}$

19. The dimensions of potential difference are
(a) $ML^2 T^{-3} A^{-1}$ (b) $MLT^{-2} A^{-1}$
(c) $ML^2 T^{-2} A$ (d) $MLT^{-2} A$

20. What are the dimensions of electrical resistance?
(a) $ML^2 T^{-2} A^2$ (b) $ML^2 T^{-3} A^{-2}$
(c) $ML^2 T^{-3} A^2$ (d) $ML^2 T^{-2} A^{-2}$

21. The dimensions of electric field are
(a) $MLT^{-3} A^{-1}$ (b) $MLT^{-2} A^{-1}$
(c) $MLT^{-1} A^{-1}$ (d) $MLT^0 A^{-1}$

22. The dimensions of magnetic field are
(a) $ML^0 T^{-1} A^{-1}$ (b) $M^0L T^{-1} A^{-1}$
(c) $MLT^{-2} A^{-1}$ (d) $ML^0T^{-2} A^{-1}$

Level B

23. The quantities L/R and RC (where L, C and R stand for inductance, capacitance and resistance respectively) have the same dimensions as those of
(a) velocity (b) acceleration
(c) time (d) force

24. The equation of state of a real gas can be expressed as $\left(P + \dfrac{a}{V^2}\right)(V - b) = cT$ where P is the pressure, V the volume, T the absolute temperature and a, b and c are constants. What are the dimensions of a?
(a) $M^0L^3T^{-2}$ (b) ML^5T^{-2}
(c) $M^0L^3T^0$ (d) $ML^{-2}T^5$

25. The equation of state for n moles of an ideal gas is

$$PV = nRT$$

where R is the universal gas constant and P, V and T have the usual meanings. What are the dimensions of R?
(a) $M^0LT^{-2} K^{-1} mol^{-1}$
(b) $ML^2 T^{-2} K^{-1} mol^{-1}$
(c) $M^0 L^2 T^{-2} K^{-1} mol^{-1}$
(d) $ML^{-2} T^{-2} K^{-1} mol^{-1}$

26. According to the quantum theory, the energy E of a photon of frequency ν is given by
$$E = h\nu$$
where h is Planck's constant. What is the dimensional formula for h?
(a) $M L^2 T^{-2}$ (b) $M L^2 T^{-1}$
(c) $M L^2 T$ (d) $M L^2 T^2$

27. What is the SI unit of Planck's constant?
(a) watt second (b) watt per second
(c) joule second (d) joule per second

28. The dimensions of Planck's constant are the same as those of
(a) energy (b) power
(c) angular frequency (d) angular momentum

29. When a wave traverses a medium, the displacement of a particle located at x at time t is given by
$$y = a \sin (bt - cx)$$
where a, b and c are constants of the wave. The dimensions of b are the same as those of
(a) wave velocity (b) amplitude
(c) wavelength (d) wave frequency

30. In Q. 28, the dimensions of $\frac{b}{c}$ are the same as those of
(a) wave velocity (b) wavelength
(c) wave amplitude (d) wave frequency

31. The van der Waal equation for n moles of a real gas is
$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$
where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are van der Waal constants. The dimensions of a are the same as those of
(a) PV (b) PV^2
(c) P^2V (d) P/V

32. In Q. 30, the dimensions of b are the same as those of
(a) P (b) V
(c) PV (d) nRT

33. In Q. 30, the dimensions of nRT are the same as those of
(a) energy (b) force
(c) pressure (d) specific heat

34. In Q. 30, the dimensional formula for ab is
(a) ML^2T^{-2} (b) ML^4T^{-2}
(c) ML^6T^{-2} (d) ML^8T^{-2}

35. If velocity (V), acceleration (A) and force (F) are taken as fundamental quantities instead of mass (M), length (L) and time (T), the dimensions of Young's modulus would be
(a) FA^2V^{-2} (b) FA^2V^{-3}
(c) FA^2V^{-4} (d) FA^2V^{-5}

36. The dimensions of permittivity (ε_0) of vacuum are
(a) $M^{-1} L^{-3} T^4 A^2$ (b) $ML^{-3} T^2 A^2$
(c) $M^{-1} L^3 T^4 A^2$ (d) $ML^3 T^2 A^2$

37. What are the dimensions of permeability (μ_0) of vacuum?
(a) $MLT^{-2} A^2$ (b) $MLT^{-2} A^{-2}$
(c) $ML^{-1} T^{-2} A^2$ (d) $ML^{-1} T^{-2} A^{-2}$

38. The dimensions of $1/\sqrt{\mu_0 \varepsilon_0}$ are the same as those of
(a) velocity (b) acceleration
(c) force (d) energy

39. What are the dimensions of magnetic flux?
(a) $ML^2 T^{-2} A^{-1}$ (b) $ML^2 T^{-2} A^{-2}$
(c) $ML^{-2} T^{-2} A^{-1}$ (d) $ML^{-2} T^{-2} A^{-2}$

40. The dimensions of self inductance are
(a) $ML^2 T^{-2} A^{-1}$ (b) $ML^2 T^{-2} A^{-2}$
(c) $ML^{-2} T^{-2} A^{-1}$ (d) $ML^{-2} T^{-2} A^{-2}$

41. The dimensions of capacitance are
(a) $M^{-1} L^{-2} TA^2$ (b) $M^{-1} L^{-2} T^2 A^2$
(c) $M^{-1} L^{-2} T^3 A^2$ (d) $M^{-1} L^{-2} T^4 A^2$

42. Frequency (n) of a tuning fork depends upon length (l) of its prongs, density (ρ) and Young's modulus (Y) of its material. Then frequency and Young's modulus will be related as
(a) $n \propto \sqrt{Y}$ (b) $n \propto Y$
(c) $n \propto \frac{1}{\sqrt{Y}}$ (d) $n \propto \frac{1}{Y}$

43. The dimensions of $\frac{1}{2} \varepsilon_0 E^2$ (ε_0 = permittivity of free space and E = electric field) are
(a) MLT^{-1} (b) ML^2T^{-2}
(c) $ML^{-1} T^{-2}$ (d) $ML^2 T^{-1}$

44. Of the following quantities, which one has dimensions different from the remaining three
(a) Energy per unit volume
(b) Force per unit area
(c) Product of voltage and charge per unit volume
(d) Angular momentum

45. If the time period t of a drop of liquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{d^a r^b s^c}$ and if $a = 1$, $c = -1$, then b is
(a) 1 (b) 2
(c) 3 (d) 4

46. A pair of physical quantities having the same dimensional formula is
(a) angular momentum and torque
(b) torque and energy
(c) entropy and power
(d) power and angular momentum

47. In the measurement of a physical quantity $X = \frac{A^2 B}{C^{1/3} D^3}$. The percentage errors introduced in the measurements of the quantities *A*, *B*, *C* and *D* are 2%, 2%, 4% and 5% respectively. Then the minimum amount of percentage of error in the measurement of *X* is contributed by:
(a) *A* (b) *B*
(c) *C* (d) *D*

48. Which of the following has the dimensions $ML^{-1}T^{-1}$?
(a) Surface tension
(b) Coefficient of viscosity
(c) Bulk modulus
(d) Angular momentum

49. Pressure gradient *dp*/*dx* is the rate of change of pressure with distance. What are the dimensions of *dp*/*dx*?
(a) $ML^{-1}T^{-1}$ (b) $ML^{-2}T^{-2}$
(c) $ML^{-1}T^{-2}$ (d) $ML^{-2}T^{-1}$

50. If *E*, *M*, *J* and G respectively denote energy, mass, angular momentum and gravitational constant, then $\frac{EJ^2}{M^5G^2}$ has the dimensions of
(a) length (b) angle
(c) mass (d) time

51. If *e*, ε_0, *h* and *c* respectively represent electronic charge, permittivity of free space, Planck's constant and speed of light, then $\frac{e^2}{\varepsilon_0 hc}$ has the dimensions of
(a) current (b) pressure
(c) angular momentum (d) angle

52. If *L*, *R*, *C* and *V* respectively represent inductance, resistance, capacitance and potential difference, then the dimensions of $\frac{L}{RCV}$ are the same as those of
(a) current (b) $\frac{1}{\text{current}}$
(c) charge (d) $\frac{1}{\text{charge}}$

53. If *E* and *B* respectively represent electric field and magnetic induction field, then the ratio $\frac{E}{B}$ has the dimensions of
(a) displacement (b) velocity
(c) acceleration (d) angle

54. If *C* and *V* respectively represent the capacitance of a capacitor and the potential difference between its plates, then the dimensions of CV^2 are
(a) ML^2T^{-2} (b) $ML^3T^{-2}I^{-1}$
(c) $ML^2T^{-1}I^{-1}$ (d) $M^0L^0T^0$

55. If *h* and *e* respectively represent Planck's constant and electronic charge, then the dimensions of $\left(\frac{h}{e}\right)$ are the same as those of
(a) magnetic field (b) electric field
(c) magnetic flux (d) electric flux

56. If energy E, velocity V and time T are chosen as the fundamental units, the dimensional formula for surface tension will be
(a) $E V^2T^{-2}$ (b) $E V^{-1}T^{-2}$
(c) $E V^{-2}T^{-2}$ (d) $E^2V^{-1}T^{-2}$

57. A gas bubble from an explosion under water oscillates with a period proportional to $P^a d^b E^c$ where P is the static pressure, d is the density of water and E is the energy of explosion. Then *a*, *b* and *c* respectively are
(a) $\frac{-5}{6}, \frac{1}{2}, \frac{1}{3}$ (b) $\frac{1}{2}, \frac{-5}{6}, \frac{1}{3}$
(c) $\frac{1}{3}, \frac{1}{2}, \frac{-5}{6}$ (d) 1, 1, 1

58. The error in the measurement of the radius of a sphere is 1%. The error in the measurement of the volume is
(a) 1% (b) 3%
(c) 5% (d) 8%

59. If the error in the measurement of the volume of a sphere is 6%, then the error in the measurement of its surface area will be
(a) 2% (b) 3%
(c) 4% (d) 7.5%

60. The percentage errors in the measurements of the length of a simple pendulum and its time period are 2% and 3% respectively. The maximum error in the value of the acceleration due to gravity obtained from these measurements is
(a) 5% (b) 1%
(c) 8% (d) 10%

61. The moment of inertia of a body rotating about a given axis is 6.0 kg m^2 in the SI system. What is the value of the moment of inertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?
(a) 2.4×10^3 (b) 2.4×10^5
(c) 6.0×10^3 (d) 6.0×10^5

62. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$ where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of
(a) resistance (b) charge
(c) voltage (d) current

63. The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula

$$\eta = \frac{\pi}{8}\frac{R^4}{l}\frac{P}{Q}$$

where R = radius of the capillary tube,
l = length of the tube,
P = pressure difference between its ends, and
Q = volume of liquid flowing per second.

Which quantity must be measured most accurately?
(a) R (b) l
(c) P (d) Q

64. The mass m of the heaviest stone that can be moved by the water flowing in a river depends on v, the speed of water, density (d) of water and the acceleration due to gravity (g). Then m is proportional to
(a) v^2 (b) v^4
(c) v^6 (d) v^8

65. The speed (v) of ripples depends upon their wavelength (λ), density (ρ) and surface tension (σ) of water. Then v is proportional to
(a) $\sqrt{\lambda}$ (b) λ
(c) $\frac{1}{\lambda}$ (d) $\frac{1}{\sqrt{\lambda}}$

66. The period of revolution (T) of a planet moving round the sun in a circular orbit depends upon the radius (r) of the orbit, mass (M) of the sun and the gravitation constant (G). Then T is proportional to
(a) $r^{1/2}$ (b) r
(c) $r^{3/2}$ (d) r^2

67. If the velocity of light (c), gravitational constant (G) and planck's constant (h) are chosen as fundamental units, the dimensions of time in the new system will be
(a) $c^{-5/2}G^2h^{-1/2}$ (b) $c^{-3/2}G^{-2}h^2$
(c) $c^2G^{-2}h^{1/2}$ (d) $c^{-5/2}G^{1/2}h^{1/2}$

68. The amplitude of a damped oscillator of mass m varies with time t as

$$A = A_0 e^{(-at/m)}$$

The dimensions of a are
(a) ML^0T^{-1} (b) M^0LT^{-1}
(c) MLT^{-1} (d) $ML^{-1}T$

69. A student measures the value of g with the help of a simple pendulum using the formula

$$g = \frac{4\pi^2 L}{T^2}$$

The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?
(a) $\Delta L = 0.5$ cm, $\Delta T = 0.5$ s
(b) $\Delta L = 0.2$ cm, $\Delta T = 0.2$ s
(c) $\Delta L = 0.1$ cm, $\Delta T = 1.0$ s
(d) $\Delta L = 0.1$ cm, $\Delta T = 0.1$ s

70. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with a uncertainty of ± 0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained from the reading is
(a) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2
(b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2
(d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

71. In a vernier callipers, one main scale division is x cm and n divisions of the vernier scale coincide with $(n - 1)$ divisions of the main scale. The least count (in cm) of the callipers is
(a) $\left(\frac{n-1}{n}\right)x$ (b) $\frac{nx}{(n-1)}$
(c) $\frac{x}{n}$ (d) $\frac{x}{(n-1)}$

72. In the relation

$$P = a\mathrm{e}^{-\frac{bx}{kT}}$$

where x is a distance, k is Boltzmann constant, T is the absolute temperature and a and b are constants, the dimensions of $\frac{a}{b}$ are the same as those of
(a) area (b) $\frac{1}{\text{area}}$
(c) volume (d) $\frac{1}{\text{volume}}$

73. The intensity I of a wave falls with distance x as

$$I = a\mathrm{e}^{-bx}$$

where a and b are constants. The dimensional formula of ab is

(a) $[ML^{-1} T^{-2}]$ (b) $[ML^2 T^{-3}]$

(c) $[ML^{-1} T^{-3}]$ (d) $[MLT^{-3}]$

74. The distance x moved by a particle in time t is given by

$$x = a\,(1 - e^{-bt})$$

The dimensions of ab are the same as those of

(a) velocity (b) momentum

(c) force (d) impulse

75. The energy E of a particle at position x at time t is given by

$$E = \frac{a}{t\,(b + x^2)}$$

where a and b are constants. The dimensional formula of a is

(a) MLT^{-1} (b) ML^2T^{-1}

(c) ML^3T^{-1} (d) ML^4T^{-1}

76. Two resistors of resistances $R_1 = (100 \pm 1)\ \Omega$ and $R_2 = (200 \pm 2)\ \Omega$ are connected in parallel. The resistance of the parallel combination is

(a) $(66.67 \pm 0.67)\,\Omega$ (b) $(66.67 \pm 0.7)\,\Omega$

(c) $(67 \pm 1)\,\Omega$ (d) $(66.7 \pm 0.7)\,\Omega$

77. The time period T of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum and g is the acceleration due to gravity. The value of L is measured to be 100.0 cm using a metre scale of least count 0.1 cm. The time for 20 oscillations is measured to be 40.0s using a stop-watch of least count 0.1s. The calculated value of g is

(a) $(9.87 \pm 0.06)\,ms^{-2}$ (b) $(9.9 \pm 0.1)\,ms^{-2}$

(c) $(9.86 \pm 0.05)\,ms^{-2}$ (d) $(9.872 \pm 0.059)\,ms^{-2}$

Answers

Level A

1. (d)	2. (b)	3. (c)	4. (c)
5. (a)	6. (b)	7. (a)	8. (d)
9. (a)	10. (d)	11. (b)	12. (a)
13. (a)	14. (b)	15. (d)	16. (c)
17. (d)	18. (b)	19. (a)	20. (b)
21. (a)	22. (d)		

Level B

23. (c)	24. (b)	25. (b)	26. (b)
27. (c)	28. (d)	29. (d)	30. (a)
31. (b)	32. (b)	33. (a)	34. (d)
35. (c)	36. (a)	37. (b)	38. (a)
39. (a)	40. (b)	41. (d)	42. (a)
43. (c)	44. (d)	45. (c)	46. (b)
47. (c)	48. (b)	49. (b)	50. (b)
51. (d)	52. (b)	53. (b)	54. (a)
55. (c)	56. (c)	57. (a)	58. (b)
59. (c)	60. (c)	61. (b)	62. (d)
63. (a)	64. (c)	65. (d)	66. (c)
67. (d)	68. (a)	69. (d)	70. (b)
71. (c)	72. (b)	73. (c)	74. (a)
75. (d)	76. (d)	77. (a)	

Solutions

Level A

1. Choices (a), (b) and (c) are units of length. Choice (d) the radian is a unit of angle in a plane.

2. Impulse = force × time

$$\frac{d\mathbf{p}}{dt} \times dt$$
$$= d\mathbf{p}$$
= change in momentum

Hence the correct choice is (b).

3. The dimensions of moment of inertia are ML^2T^0 and of moment of force are $ML^2\,T^{-2}$. All other pairs in (a), (b) and (d) have identical dimensions.

4. The viscous force acting on a spherical body of radius r moving with a speed v in a fluid of coefficient of viscosity η is given by

$$F = 6\pi\,\eta\,r\,v \quad \text{or} \quad \eta = \frac{F}{6\pi r v}$$

∴ Dimensions of η

$$= \frac{\text{dimension of } F}{\text{dimension of } r \times \text{dimension of } v}$$

$$= \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$$

Hence the correct choice is (c).

5. Surface tension = force/length = $MLT^{-2}/L = ML^0T^{-2}$. Hence the correct choice is (a).
6. From Newton's law of gravitation, the force of attraction F between two bodies of masses m_1 and m_2, separated by a distance r is given by

$$F = G\,\frac{m_1m_2}{r^2};$$

G = gravitational constant

or $$G = \frac{Fr^2}{m_1m_2}$$

$$\therefore \quad \text{Unit of } G = \frac{\text{unit of } F \times \text{unit of } r^2}{\text{unit of } m_1m_2}$$

$$= \frac{\text{newton} \times (\text{metre})^2}{(\text{kg})^2} = \text{N m}^2\,\text{kg}^{-2}$$

Hence, the correct choice is (b).
7. The coefficient of thermal conductivity is defined as the rate of flow of heat energy per unit area of cross-section per unit temperature gradient. The dimensions of rate of flow of heat are $ML^2\,T^{-2}/T = ML^2\,T^{-3}$. The dimensions of area are L^2 and the dimensions of temperature gradient are = dimensions of temperature/dimension of length = KL^{-1}. Hence the dimensions of the coefficient of thermal conductivity are

$$\frac{ML^2\,T^{-3}}{L^2 \times KL^{-1}} = MLT^{-3}\,K^{-1}$$

Thus, the correct choice is (a).
8. From Stefan's law, the total energy emitted per second by a unit area of a black body is proportional to the fourth power of its absolute temperature, i.e.

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4$$

where σ is the Stefan's constant. Thus

$$\sigma = \frac{E}{T^4}$$

$\therefore$ SI unit of σ

$$= \frac{\text{SI unit of energy per second}}{\text{SI unit of area} \times (\text{SI unit of temperature})^4}$$

$$= \frac{\text{watt}}{(\text{metre})^2 \times (\text{kelvin})^4} = \text{W m}^{-2}\,\text{K}^{-4}$$

Hence the correct choice is (d).
9. The dimensions of energy are $ML^2\,T^{-2}$. The dimensions of pressure, momentum and power are $ML^{-1}\,T^{-2}$, MLT^{-1} and $ML^2\,T^{-3}$ respectively. Thus the correct choice is (a).
10. $ML^{-1}\,T^{-2}$ are the dimensions of force per unit area. Out of the four choices, stress is the only quantity that is force per unit area. Hence the correct choice is (d).
11. The angular momentum **L** of a particle with respect to point whose position vector is **r** is given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **p** is the linear momentum of the moving particle.

$\therefore$ Dimensions of **L** = dimension of **r** × dimensions of **p** = $L \times MLT^{-1} = ML^2T^{-1}$

Thus the correct choice is (b).
12. Since $G = \dfrac{Fr^2}{m_1m_2}$, the dimensions of G are

$$(G) = \frac{\text{dimensions of } F \times \text{dimensions of } r^2}{\text{dimensions of } m_1m_2}$$

$$= \frac{MLT^{-2} \times L^2}{M^2} = M^{-1}L^3T^{-2}$$

Thus, the correct choice is (a).
13. The dimensions of the two sides of proportionality are

$$T = M^a\,L^b\,(LT^{-2})^c$$

where LT^{-2} are the dimensions of g.

$$\therefore \quad T = M^a\,L^b\,L^c\,T^{-2c} = M^a\,L^{b+c}\,T^{-2c}$$

Equating the powers of dimensions on both sides, we have $a = 0$, $b + c = 0$ and $-2c = 1$

which give $c = -\dfrac{1}{2}$, $b = \dfrac{1}{2}$ and $a = 0$.

Thus, the correct chioce is (a).
14. The dimensions of the two sides of proportionality are

$$L^3 = L^{2\alpha}(LT^{-1})^{\beta}T^{\gamma} = L^{2\alpha+\beta}T^{\gamma-\beta}$$

Equating the powers of dimensions on both sides, we have

$$2\alpha + \beta = 3$$
$$\gamma - \beta = 0$$

which give $\beta = \gamma$ and $\alpha = \dfrac{1}{2}(3 - \beta)$, i.e. $\alpha \neq \beta = \gamma$.

Thus, the correct choice is (b).
15. Squaring both sides of the given relation, we get

$$n^2 = \frac{p^2}{4\,l^2}\cdot\frac{F}{m} \quad \text{or} \quad m = \frac{p^2F}{4\,l^2n^2}$$

∴ dimensions of m

$$= \frac{\text{dimensions of } F}{\text{dimensions of } l^2 \times \text{dimensions of } n^2}$$

($\because$ p is a dimensionless number)

$$= \frac{MLT^{-2}}{L^2 \times (T^{-1})^2} = ML^{-1}T^0$$

Hence, the correct choice is (d).

16. The heat energy content H of a body of mass m at temperature θ is given by $H = ms\theta$ where s is the specific heat. Therefore

$$s = \frac{H}{m\theta}$$

∴ Dimensions of s

$$= \frac{\text{dimensions of heat energy}}{\text{dimension of mass} \times \text{dimension of temperature}}$$

$$= \frac{ML^2T^{-2}}{M \times K} = M^0L^2T^{-2}K^{-1}$$

Thus, the correct choice is (c).

17. Latent heat L is the amount of heat energy H required to change the state of a unit mass without producing any change in temperature. Thus

$$L = \frac{H}{m}$$

∴ Dimensions of $L = \frac{ML^2T^{-2}}{M}$

$$= L^2T^{-2} = M^0L^2T^{-2}$$

Thus, the correct choice is (d).

18. According to the law of equipartition of energy, the energy per degree of freedom of a gas atom or molecule at a temperature θ kelvin is given by

$$E = \frac{1}{2}k\theta \quad \text{or} \quad k = \frac{2E}{\theta}$$

where k is the Boltzmann's constant.

∴ Dimensions of $k = \frac{\text{dimensions of } E}{\text{dimension of } \theta}$

$$= \frac{ML^2T^{-2}}{K} = ML^2T^{-2}K^{-1}$$

19. The potential difference V between two points is the amount of work done in moving a unit charge from one point to the other.

Thus, $V = \frac{\text{work done}}{\text{charge moved}} = \frac{W}{q}$

∴ Dimensions of $V = \frac{ML^2T^{-2}}{Q} = ML^2T^{-2}Q^{-1}$

$$= ML^2T^{-3}A^{-1} \quad (\because Q = AT)$$

Hence, the correct choice is (a).

20. From Ohm's law, resistance R is given by

$$R = \frac{\text{potential difference}}{\text{current}}$$

∴ Dimensions of $R = \frac{ML^2T^{-3}A^{-1}}{A} = ML^2T^{-3}A^{-2}$

Thus, the correct choice is (b).

21. Force F experienced by a charge q in an electric field E is given by

$$F = qE \text{ or } E = \frac{F}{q}$$

∴ Dimensions of $E = \frac{\text{dimensions of } F}{\text{dimensions of } Q} = \frac{MLT^{-2}}{AT}$

$$= MLT^{-3}A^{-1}.$$

22. The force F, experienced by a charge q moving with speed v perpendicular to the direction of a uniform magnetic induction field B is given by

$$F = qvB \text{ or } B = \frac{F}{qv}$$

∴ Dimensions of $B = \frac{MLT^{-2}}{Q \times LT^{-1}} = ML^0T^{-1}Q^{-1}$

$$= ML^0T^{-2}A^{-1} \quad (\because Q = AT)$$

Hence, the correct choice is (d).

Level B

23. L/R is the time constant of an L-R circuit and CR is the time constant of a C-R circuit. The dimension of the time constant is the same as that of time. Hence the correct choice is (c).

24. From the principle of homogeneity of dimensions, the dimensions of $\frac{a}{V^2}$ must be the same as those of P. Therefore dimensions of a = dimensions of $P \times$ dimensions of V^2

$= ML^{-1}T^{-2} \times (L^3)^2 = ML^{-1}T^{-2} \times L^6 = ML^5T^{-2}$.

Thus, the correct choice is (b).

25. Since $R = \frac{PV}{nT}$, the dimensions of R are given by

$$(R) = \frac{\text{dimensions of } P \times \text{dimensions of } V}{\text{dimensions of } n \times \text{dimensions of } T}$$

$$= \frac{ML^{-1}T^{-2} \times L^3}{\text{mol} \times K} = ML^2T^{-2}\,\text{mol}^{-1}K^{-1}$$

Thus the correct choice is (b).

26. Dimensions of $h = \dfrac{\text{dimension of } E}{\text{dimension of } \nu}$

$$= \frac{ML^2T^{-2}}{T^{-1}} = ML^2\,T^{-1}$$

Thus, the correct choice is (b).

27. Unit of $h = \dfrac{\text{unit of } E}{\text{unit of } \nu} = \dfrac{\text{joule}}{(\text{second})^{-1}}$ = joule second.

Thus, the correct choice is (c).

28. The correct choice is (d).

29. Since the argument of a sine function (or any trigonometric function) must be dimensionless, bt and cx are dimensionless. Since bt is dimensionless, the dimensions of b = dimensions of $1/t = T^{-1}$, which are the dimensions of frequency. Hence, the correct choice is (d).

30. Dimensions of bt = dimensions of cx, as they are both dimensionless.

$\therefore$ Dimensions of $\dfrac{b}{c}$ = dimensions of $\dfrac{x}{t} = \dfrac{L}{T} = LT^{-1}$.

Hence, the correct choice is (a).

31. From the principle of homogeneity, the dimensions of $\dfrac{a}{V^2}$ must be the same as those of P, i.e. dimensions of $\dfrac{a}{V^2}$ = dimensions of P

$\therefore$ dimensions of a = dimensions of PV^2. Hence, the correct choice is (b).

32. The correct choice is (b).

33. The dimensions of nRT = dimensions of PV

$$= ML^{-1}\,T^{-2} \times L^3 = ML^2\,T^{-2}$$

which are the dimensions of energy. Hence, the correct choice is (a)

34. The dimensions of $\dfrac{ab}{V^2}$ are the same as those of PV.

$\therefore$ Dimensions of ab = dimensions of $(PV) \times V^2 = ML^2\,T^{-2} \times L^6 = ML^8\,T^{-2}$

Hence, the correct choice is (d).

35. Dimensions of Young's modulus Y are $ML^{-1}\,T^{-2}$. The dimension of V, A and F in terms of M, L and T are

$$(V) = LT^{-1},\ (A) = LT^{-2}$$

and $(F) = MLT^{-2}$

Let $(Y) = (V^a\,A^b\,F^c)$

Putting dimensions of Y, V, A and F. we have

$$(ML^{-1}\,T^{-2}) = (LT^{-1})^a \times (LT^{-2})^b \times (MLT^{-2})^c$$

or $M^1\,L^{-1}\,T^{-2} = M^c\,L^{a+b+c}\,T^{-a-2b-2c}$

Equating powers of M, L and T we have

$$c = 1,\ a + b + c = -1$$

and $-a - 2b - 2c = -2$

which give $a = -4$, $b = 2$ and $c = 1$.

Thus $(Y) = (FA^2V^{-4})$

Thus, the correct choice is (c).

36. According to Coulomb's law of electrostatics, force F between two charges q_1 and q_2 a distance r apart in vacuum, is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

or

$$\varepsilon_0 = \frac{1}{4\pi F} \cdot \frac{q_1 q_2}{r^2}$$

$\therefore$ Dimensions of $\varepsilon_0 = \dfrac{Q^2}{MLT^{-2} \times L^2}$

$$= M^{-1}\,L^{-3}\,T^2\,Q^2 = M^{-1}L^{-3}T^4A^2 \quad \left(\because A = \frac{Q}{T}\right)$$

The correct choice is (a).

37. The force per unit length between two long wires carrying currents I_1 and I_2 a distance r apart in vacuum, is given by

$$f = \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2}{r} \quad \text{or} \quad \mu_0 = \frac{4\pi r f}{I_1 I_2}$$

$\therefore$ Dimensions of $\mu_0 = \dfrac{L \times MLT^{-2} \times L^{-1}}{A^2}$

$$= MLT^{-2}\,A^{-2}$$

Therefore, the correct choice is (b).

38. Dimensions of

$$\frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = \frac{1}{\left(MLT^{-2}\,A^{-2} \times M^{-1}L^{-3}T^4A^2\right)^{\frac{1}{2}}}$$

$$= \frac{1}{\left(L^{-2}T^2\right)^{\frac{1}{2}}} = LT^{-1}$$

which are the dimensions of velocity. Hence the correct choice is (a).

39. The magnetic flux ϕ linked with a circuit of area A in a magnetic induction field B is given by

$$\phi = BA\cos\theta$$

where θ is the angle between the field and area vectors.

$\therefore$ Dimensions of ϕ = dimensions of BA

($\because$ cos θ is dimensionless)

$= ML^0T^{-2}A^{-1} \times L^2 = ML^2T^{-2}A^{-1}$

Thus, the correct choice is (a).

40. The self inductance L of a coil in which the current varies at a rate $\frac{dI}{dt}$ is given by

$$e = -L\frac{dI}{dt}$$

where e is the e.m.f. induced in the coil. Now, the dimensions of e.m.f. are the same as those of potential difference, namely, $ML^2\ T^{-3}\ A^{-1}$.

Now, $$L = -\frac{e}{\frac{dI}{dt}}$$

$\therefore$ Dimensions of L

$$= \frac{\text{dimensions of } e}{\text{dimensions of } I / \text{dimensions of } t}$$

$$= \frac{ML^2T^{-3}A^{-1}}{A/T} = ML^2\ T^{-2}\ A^{-2}$$

Thus, the correct choice is (b).

41. When a capacitor of capacitance C is charged to a potential difference V, the charge Q on the capacitor plates is given by

$$Q = CV \text{ or } C = \frac{Q}{V}$$

$\therefore$ Dimensions of $C = \frac{\text{dimensions of } Q}{\text{dimensions of } V}$

$$= \frac{AT}{ML^2T^{-3}A^{-1}} = M^{-1}\ L^{-2}\ T^4\ A^2$$

Hence, the correct choice is (d).

42. Let $n \propto l^a \rho^b Y^c$

Putting dimensions of all the quantities, we have

$$(T^{-1}) \propto L^a\ (ML^{-3})^b\ (ML^{-1}\ T^{-2})^c$$

Equating powers of M, L and T on both sides, we get

$$b + c = 0,\ a - 3b - c = 0 \text{ and } -2c = -1$$

which give $a = -1$, $b = -\frac{1}{2}$ and $c = \frac{1}{2}$. Thus

$$n \propto l^{-1}\ \rho^{1/2}\ Y^{1/2}$$

Hence, the correct choice is (a).

43. We know that

$$F = \frac{q_1\ q_2}{(4\pi\varepsilon_0)r^2} \text{ and } E = \frac{F}{q}$$

$$\therefore \quad \varepsilon_0 = \frac{q_1\ q_2}{4\pi F\ r^2}$$

Hence $$\frac{1}{2}\varepsilon_0 E^2 = \frac{q_1\ q_2}{8\pi F\ r^2} \times \frac{F^2}{q^2} = \frac{q_1\ q_2}{8\pi\ q^2} \times \frac{F}{r^2}$$

$\therefore$ Dimensions of $\frac{1}{2}\varepsilon_0 E^2$ = dimensions of $\frac{F}{r^2}$

$$= \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Hence, the correct choice is (c).

44. Energy per unit volume, force per unit area and product of voltage and charge density all have dimensions of ML^2T^{-2}, but the dimensions of angular momentum are ML^2T^{-1}. Hence, the correct choice is (d).

45. Given $t = d^{a/2}\ r^{b/2}\ s^{c/2}$. Substituting dimensions, we have

$$(T) = (ML^{-3})^{a/2}\ (L)^{b/2}\ (MT^{-2})^{c/2}$$
$$= M^{(a+c)/2} \cdot L^{(-3a/2 + b/2)}\ T^{-c}$$

Equating powers of L, we have, $-\frac{3}{2}a + \frac{b}{2} = 0$. Given $a = 1$.

$$\therefore \quad -\frac{3}{2} + \frac{b}{2} = 0 \quad \text{or} \quad b = 3,$$

which is choice (c).

46. Both torque and energy have the dimensions of force $\times$ distance. Hence the correct choice is (b).

47. Given $X = \frac{A^2\ B}{C^{1/3}\ D^3}$

Taking logarithm of both sides, we have

$$\log X = 2 \log A + \log B - \frac{1}{3} \log C - 3 \log D$$

Partially differentiating, we have

$$\frac{\Delta x}{x} = 2\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{1}{3}\frac{\Delta C}{C} - 3\frac{\Delta D}{D}$$

Percentage error in $A = 2\frac{\Delta A}{A} = 2 \times 2\% = 4\%$

Percentage error in $B = \frac{\Delta B}{B} = 2\%$

Percentage error in $C = \frac{1}{3}\frac{\Delta C}{C} = \frac{1}{3} \times 4\%$

$$= \frac{4}{3}\%$$

Percentage error in $D = 3\,\frac{\Delta D}{D} = 3 \times 5\% = 15\%$

We find that the minimum percentage error is contributed by C. Hence the correct choice is (c).

48. The correct choice is (b)
49. The correct choice is (b)
50. Dimensions of J and G are ML^2T^{-1} and $M^{-1}L^3\,T^{-2}$ respectively. The correct choice is (b).
51. Dimensions of ε_0 and h are $M^{-1}L^{-3}T^4A^2$ and ML^2T^{-1} respectively. The correct choice is (d).
52. RC has the dimensions of time (T). V has the dimensions of emf which has the dimensions of $L\,\frac{dI}{dt}$. The correct choice is (b).
53. The force **F** on a particle of charge q moving with a velocity **v** in **E** and **B** fields is given by
$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$
Hence the dimensions of E are the same as those of vB. The correct choice is (b).
54. Energy stored in a capacitor of capacitance C having a potential difference V between its plates is given by
$$U = \frac{1}{2}CV^2$$
Hence, the dimensions of CV^2 = dimensions of energy. Hence the correct choice is (a).
55. Dimensions of $\left(\frac{h}{e}\right) = \frac{(ML^2T^{-1})}{AT} = ML^2\,T^{-2}\,A^{-1}$

Dimensions of $B = MT^{-2}\,A^{-1}$

Magnetic flux = $B \times$ area. The correct choice is (c).
56. Let surface tension $\sigma = E^a V^b T^c$. Using the dimensions of σ, E, V and T and equating powers of M, L and T, find the values of a, b and c. The correct choice is (c).
57. The correct choice is (a).
58. $V = \frac{4}{3}\pi r^3$. Taking logarithm of both sides, we have
$$\log V = \log 4 + \log \pi + 3 \log r - \log 3$$
Differentiating, we get
$$\frac{\delta V}{V} = 3\,\frac{\delta r}{r} = 3 \times 1\% = 3\%$$
59. $\frac{\delta V}{V} = 3\,\frac{\delta r}{r}$ or $6\% = 3\,\frac{\delta r}{r}$ or $\frac{\delta r}{r} = 2\%$.

Now surface area $s = 4\pi r^2$ or $\log s = \log 4\pi + 2 \log r$
$$\therefore \quad \frac{\delta s}{s} = 2\,\frac{\delta r}{r} = 2 \times 2\% = 4\%.$$
60. $T = 2\pi\sqrt{\frac{l}{g}}$ or $g = 4\pi^2\,\frac{l}{T^2}$

or $\log g = \log(4\pi^2) + \log l - 2 \log T$. The maximum error in g is
$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\,\frac{\Delta T}{T} = 2\% + 2 \times 3\% = 8\%$$
61. The dimensions of moment of inertia are (ML^2). We have
$$n_1(u_1) = n_2(u_2)$$
or $\quad n_1(M_1L_1^2) = n_2(M_2\,L_2^2)$
$$\therefore \quad n_2 = \frac{n_1(M_1L_1^2)}{(M_2L_2^2)} = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2$$
Given $n_1 = 6.0$, $M_1 = 1$ kg, $L_1 = 1$ m, $M_2 = 10$ g and $L_2 = 5$ cm. Therefore,
$$n_2 = 6.0 \times \left(\frac{1\text{ kg}}{10\text{ g}}\right) \times \left(\frac{1\text{ m}}{5\text{ cm}}\right)^2$$
$$= 6.0 \times \left(\frac{1000\text{ g}}{10\text{ g}}\right) \times \left(\frac{100\text{ cm}}{5\text{ cm}}\right)^2$$
$$= 6.0 \times 100 \times (20)^2 = 2.4 \times 10^5$$
62. The capacitance of a parallel plate capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.
$$\therefore \text{ Dimension of } \varepsilon_0 L\,\frac{\Delta V}{\Delta t}$$
$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$
$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because Q = CV)$$
Hence the correct choice is (d).
63. The correct choice is (a).

The maximum permissible error in η is given by the relation
$$\frac{\Delta\eta}{\eta} = 4\,\frac{\Delta R}{R} + \frac{\Delta l}{l} + \frac{\Delta P}{P} + \frac{\Delta Q}{Q}$$
It is clear that the error in the measurement of R is magnified four times on account of the occurrence of R^4 in the formula. Hence the radius (R) of the capillary tube must be measured most accurately. Thus the quantity which is raised to the highest power needs the most accurate measurement.
64. Take $m \propto v^a\,d^b\,g^c$ and show that $a = 6$.
65. Take $v \propto \lambda^a\,\rho^b\,\sigma^c$ and show that $a = -\frac{1}{2}$.

66. Take $T \propto r^a M^b G^c$ and show that $a = \frac{3}{2}$.

67. The correct choice is (d).

68. The exponent is a dimensionless number. Hence at/m is dimensionless. Therefore,

$$\text{Dimension of } a = \frac{\text{dimension of } m}{\text{dimension of } t} = \frac{[\text{M}]}{[\text{T}]} = [\text{M}\,\text{L}^0\,\text{T}^{-1}]$$

69. The proportionate error in the measurement of g is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Hence Δg will be minimum if ΔL and ΔT are minimum. Thus the correct choice is (d).

70. $$Y = \frac{FL}{Al} = \frac{4MgL}{\pi d^2 l} \qquad (1)$$

where

M = 1.0 kg (exact), $g = 9.8\ \text{ms}^{-2}$ (exact)

L = 2 m (exact), $l = 0.8\ \text{mm} = 0.8 \times 10^{-3}$ m

$\Delta l = \pm 0.05$ mm, $d = 0.4\ \text{mm} = 0.4 \times 10^{-3}$ m

$\Delta d = \pm 0.01$ mm

Substituting the values of M, g, L, d and l in Eq. (1) we get

$$Y = 2.0 \times 10^{11}\ \text{Nm}^{-2}$$

From Eq. (1) the proportionate uncertainty in Y is given by

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

Since the values of M, g and L are exact, $\Delta M = 0$, $\Delta g = 0$ and $\Delta L = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l} = \frac{2 \times 0.01\,\text{mm}}{0.4\,\text{mm}} + \frac{0.05\,\text{mm}}{0.8\,\text{mm}}$$

$$= 0.05 + 0.0625 = 0.1125$$

$\therefore\ \Delta Y = 0.1125 \times Y = 0.1125 \times 2.0 \times 10^{11}$

$= 0.225 \times 10^{11}\ \text{Nm}^{-2}$

Since the value of Y is correct only up to the first decimal place, the value of ΔY must be rounded off to the first decimal place. Thus $\Delta Y = 0.2 \times 10^{11}\ \text{Nm}^{-2}$. Therefore, the result of the experiment is

$$Y + \Delta Y = (2.0 \pm 0.2) \times 10^{11}\ \text{Nm}^{-2}$$

Hence the correct choice is (b).

71. Vernier constant = value of 1 M.S.D – value of 1 V.S.D.

Now n V.S.D = $(n-1)$ M.S.D = $(n-1)\,x$ cm

$$\therefore \quad 1 \text{ V.S.D} = \left(\frac{n-1}{n}\right) x \text{ cm}$$

$$\therefore \quad \text{V.C.} = x \text{ cm} - \left(\frac{n-1}{n}\right) x \text{ cm} = \frac{x}{n} \text{ cm}$$

Hence the correct choice is (c).

72. Since the exponential function is dimensionless,

$$[a] = [P] = [\text{ML}^{-1}\text{T}^{-2}]$$

Since the exponent is also dimensionless,

$$[b] = \frac{[k\text{T}]}{[x]} = \frac{[\text{ML}^2\text{T}^{-2}\text{K}^{-1}] \times [\text{K}]}{[\text{L}]}$$

$$= [\text{MLT}^{-2}]$$

$$\therefore \quad \left[\frac{a}{b}\right] = \frac{[\text{ML}^{-1}\text{T}^{-2}]}{[\text{MLT}^{-2}]} = \text{L}^{-2}$$

So the correct choice is (b).

73. The intensity of a wave is defined as the rate at which the energy crosses a unit area held normal to it. Hence

$$I = \frac{\text{energy}}{\text{area} \times \text{time}} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^2] \times [\text{T}]}$$

$$= [\text{MT}^{-3}]$$

$$[a] = [I] = [\text{MT}^{-3}]$$

$$[b] = \left[\frac{1}{x}\right] = \left[\frac{1}{\text{L}}\right] = [\text{L}^{-1}]$$

$$\therefore \quad [ab] = [\text{MT}^{-3}] \times [\text{L}^{-1}]$$

$= [\text{ML}^{-1}\text{T}^{-3}]$, which is choice (c).

74. $[a] = [x] = [\text{L}]$

$[b] = [t^{-1}] = [\text{T}^{-1}]$

$\therefore \quad [ab] = [\text{L}] \times [\text{T}^{-1}] = [\text{LT}^{-1}]$

So the correct choice is (a).

75. $[b] = [x^2] = [\text{L}^2]$

$a = Et\,[b + x^2]$

$\therefore \quad [a] = [E] \times [t] \times [b]$

$= [\text{ML}^2\text{T}^{-2}] \times [\text{T}] \times \text{L}^2]$

$= [\text{ML}^4\text{T}^{-1}]$, which is choice (d)

76. Refer also to Example 32 on page 1.8.

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{100 \times 200}{100 + 200} = 66.67\ \Omega$$

From $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$, we get

$$\frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \quad \Delta R_p = \left(\frac{R_p}{R_1}\right)^2 \Delta R_1 + \left(\frac{R_p}{R_2}\right)^2 \Delta R_2$$

$$= \left(\frac{66.67}{100}\right)^2 \times 1 + \left(\frac{66.67}{200}\right)^2 \times 2.$$

$$= 0.444 + 0.222$$

$$= 0.666$$

$$= 0.7\ \Omega$$

$\therefore \quad R_p = 66.7\ \Omega$ and the result with appropriate significant figure is written as

$$R_p = (66.7 + 0.7)\Omega$$

So the correct choice is (d).

77. $T = 2\pi\sqrt{\frac{L}{g}} \quad \Rightarrow \quad g = \frac{4\pi^2 L}{T^2}.$

If the pendulum completes n oscillations in t seconds, then $T = \frac{t}{n}$. In terms of measured quantities, we have

$$g = \frac{4\pi^2 L n^2}{t^2} \qquad (1)$$

Given $L = 1.0$ m, $n = 20$ and $t = 40.0$s. Substituting these values in (1) we get $g = 9.872$ ms^{-2}. From (1), we get

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta t}{t}$$

$$= \frac{0.1\,\text{cm}}{100.0\,\text{cm}} + \frac{2 \times 0.1\,\text{s}}{40.0\,\text{s}}$$

$$= 0.001 + 0.005 = 0.006$$

$$\therefore \quad \Delta g = 0.006 \times 9.872 = 0.059 \simeq 0.06\ \text{ms}^{-2}$$

Therefore, the value of g is written as

$$g = (9.87 \pm 0.06)\ \text{ms}^{-2}$$

The correct choice is (a).

2 SECTION Multiple Choice Questions Based on Passage

Questions 1, 2 and 3 are based on the following passage.

Passage I

In the study of physics, we often have to measure the physical quantities. The numerical value of a measured quantity can only be approximate as it depends upon the least count of the meaasuring instrument used. The number of significant figures in any measurement indicates the degree of precision of that measurement. The importance of significant figures lies in calculation. A mathematical calculation cannot increase the precision of a physical measurement. Therefore, the number of significant figures in the sum or product of a group of numbers cannot be greater than the number that has the least number of significant figures. A chain cannot be stronger than its weakest link.

1. A bee of mass 0.000087 kg sits on a flower of mass 0.0123 kg. What is the total mass supported by the stem of the flower upto appropriate significant figures?
   (a) 0.012387 kg (b) 0.01239 kg
   (c) 0.0124 kg (d) 0.012 kg
2. The radius of a uniform wire is $r = 0.021$ cm. The value π is given to be 3.142. What is the area of cross-section of the wire up to appropriate significant figures.
   (a) 0.0014 cm^2 (b) 0.00139 cm^2
   (c) 0.001386 cm^2 (d) 0.0013856 cm^2
3. A man run 100.5 m is 10.3 s. His average speed up to appropriate significant figures is
   (a) 9.76 ms^{-1} (b) 9.757 ms^{-1}
   (c) 9.7573 ms^{-1} (d) 9.8 ms^{-1}

Solutions

1. Total mass = 0.000087 + 0.0123 = 0.012387 kg. The mass of the bee is accurate up to sixth decimal place in kg, whereas the mass of the flower is accurate only upto the fourth decimal place. Hence the sum must be rounded off to the fourth decimal place. Therefore the correct choice is (c).
2. $A = \pi r^2 = 3.142 \times (0.021)^2 = 0.00138562$ cm^2. Now, there are only two significant figures in

0.021 cm. Hence the result must be rounded off to two significant figure as $A = 0.0014\ \text{cm}^2$, which is choice (a).

3. Average speed = $\dfrac{100.5\ \text{m}}{10.3\ \text{s}} = 9.7573\ \text{ms}^{-1}$

The distance has four significant figures but the time has only three. Hence the result must be rounded off to three significant figure to $9.76\ \text{ms}^{-1}$. Thus the correct choice is (a).

SECTION 3 Assertion-Reason Type Questions

In the following questions, **Statement-1 (Assertion)** is followed by **Statement-2 (Reason)**. Each question has the following four options out of which only **one** choice is correct.

(a) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true but Statement-2 is *not* the correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

1. **Statement-1**

   The order of accuracy of measurement depends on the least count of the measuring instrument.

   Statement-2

   The smaller the least count the greater is the number of significant figures in the measured value.

2. **Statement-1**

   The dimensional method cannot be used to abtain the dependence of the work done by a force **F** on the angle θ between force **F** and displacement $\boldsymbol{x}$.

   Statement-2

   All trignometric functions are dimensionless.

3. **Statement-1**

   The mass of an object is 13.2 kg. In this measurement there are 3 significant figures.

   Statement-2

   The same mass when expressed in grams as 13200 g has five significant figures.

Solutions

1. The correct choice is (b).
2. Work done is $W = Fx \cos\theta$. Since $\cos\theta$ is dimensionless, the dependence of W on θ cannot be determined by the dimensional method. Hence the correct choice is (a)
3. The correct choice is (c). The degree of accuracy (and hence the number of significant figures) of a measurement cannot be increased by changing the unit.

SECTION 4 Previous Years' Questions from AIEEE, IIT-JEE, JEE (Main) and JEE (Advanced) (with Complete Solutions)

1. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of free space and E = electric field) are

   (a) MLT^{-1} (b) ML^2T^{-2}

   (c) $\text{ML}^{-1}\text{T}^{-2}$ (b) ML^2T^{-1} **[2000]**

2. A quantity X is given by $\varepsilon_0 L \dfrac{\Delta V}{\Delta t}$ where is the ε_0 is the permittivity of free space, L is the length, ΔV is a potential difference and is Δt a time interval. The dimensional formula for X is the same as that of

   (a) resistance (b) charge

   (c) voltage (d) current **[2001]**

3. The dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where the symbols have their usual meaning are
(a) $[L^{-1}T]$ (b) $[L^2T^2]$
(c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$ **[2003]**
4. The physical quantities not having the same dimensions are
(a) torque and work
(b) momentum and Planck's constant
(c) stress and Young's modulus
(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$ **[2003]**
5. In the expression
$$P = \left(\frac{\alpha^2}{\beta}\right)e^{-\alpha Z/k\theta}$$
P is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula for β is
(a) $[M L^3 \; T^{-2}]$ (b) $[M L^2 \; T^{-2}]$
(c) $[M L \; T^{-1}]$ (d) $[M^0 L^2 \; T^{-1}]$ **[2004]**
6. Which of the following denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?
(a) H/m^2 (b) Weber (Wb)
(c) Wb/m^2 (d) Henry(H) **[2006]**
7. A student measure the value of g with the help of a simple pendulum susing the formula
$$g = \frac{4\pi^2 L}{T^2}$$
The errors in the measurements of L and T are ΔL and ΔT respectively. In which of the following cases is the error in the value of g the minimum?
(a) ΔL = 0.5 cm, ΔT = 0.5 s
(b) ΔL = 0.2 cm, ΔT = 0.2 s
(c) ΔL = 0.1 cm, ΔT = 1.0 s
(d) ΔL = 0.1 cm, ΔT = 0.1 s **[2006]**
8. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with a uncertainty of ± 0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained from the reading is
(a) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2
(b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2
(d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2 **[2007]**
9. In a vernier callipers, one main scale division is x cm and n divisions of the vernier scale coincide with $(n - 1)$ divisions of the main scale. The least count (in cm) of the callipers is
(a) $\left(\frac{n-1}{n}\right)x$ (b) $\frac{nx}{(n-1)}$
(c) $\frac{x}{n}$ (d) $\frac{x}{(n-1)}$ **[2007]**
10. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using a simple pendulum. They use different lengths of the pendulum and/ or record time for different number of oscillations. The observations are shown in the table.
Least count for length = 0.1 cm
Least count for time = 0.1 s

Student	*Length of the pendulum (cm)*	*Number of oscillations (n)*	*Total time for (n) oscillations (s)*	*Time period (s)*
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	6.0

If E_I, E_{II} and E_{III} are the percentage arrors in g. i.e. $\left(\frac{\Delta g}{g} \times 100\right)$ for student I, II and III, respectively,
(a) $E_I = 0$ (b) E_I is minimum
(c) $E_I = E_{II}$ (d) E_{II} is minimum
[2008]
11. Two full turns of the circular scale of screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of –0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is
(a) 3.38 mm (b) 3.32 mm
(c) 3.73 mm (d) 3.67mm **[2008]**
12. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as
(a) 17.56 kg ms^{-1} (b) 17.57 kg ms^{-1}
(c) 17.6 kg ms^{-1} (d) 17.565 kg ms^{-1}
[2008]
13. The dimension of magnetic field in M, L, T and C (Coulomb) is given as
(a) $MT^{-1} C^{-1}$ (b) $MT^{-2} C^{-1}$
(c) $MLT^{-1} C^{-1}$ (d) $MT^{-2} C^{-2}$ **[2008]**

14. In an experiment, the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5º), then the least count of the instrument is :
    (a) one degree (b) half degree
    (c) one minute (d) half minute **[2009]**
15. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
    (a) 5, 1, 2 (b) 5, 1, 5
    (c) 5, 5, 2 (d) 4, 4, 2 **[2010]**
16. A Vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is
    (a) 0.02 mm (b) 0.05 mm
    (c) 0.1 mm (d) 0.2 mm **[2010]**
17. A screw gauge gives the following reading when used to measure the diameter of a wire. Main scale reading : 0 mm
    Circular scale reading : 52 divisions
    Given that 1 mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is:
    (a) 0.052 cm (b) 0.026 cm
    (c) 0.005 cm (d) 0.52 cm **[2011]**
18. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0. 5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is
    (a) 0.9% (b) 2.4%
    (c) 3.1% (d) 4.2% **[2011]**
19. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension ℓ = 0.25 mm in the length of the wire is observed. Quantities d and ℓ are measured using a screw gauge and a micrometer, respectively. They have same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement
    (a) due to errors in the measurement of d and ℓ are the same.
    (b) due to errors in the measurement of d is twice that due to the error in the measurement of ℓ.
    (c) due to errors in the measurement of ℓ is twice that due to the error in the measurement of d.
    (d) due to the error in the measurement of d is four times that due to the error in the measurement of ℓ. **[2012]**
20. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is:
    (a) 5.112 cm (b) 5.124 cm
    (c) 5.136 cm (d) 5. 148 cm **[2013]**
21. The physical quantities not having the same dimensions are
    (a) torque and work
    (b) momentum and Planck's constant
    (c) pressure and Young's modulus
    (d) speed and $(\mu_0 \in_0)^{-1/2}$ **[2014]**
22. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m, the gas in the tube is
    (**Useful information:** $\sqrt{167RT}$ = 640 $J^{1/2}$ $mole^{-1/2}$; $\sqrt{140RT}$ = 590 $J^{1/2}$ $mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)
    (a) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$
    (b) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$
    (c) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$
    (d) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$ **[2014]**

Answers

1. (c)	2. (d)	3. (c)	4. (b)
5. (a)	6. (d)	7. (d)	8. (b)
9. (c)	10. (b)	11. (a)	12. (c)
13. (a)	14. (c)	15. (a)	16. (d)
17. (a)	18. (c)	19. (a)	20. (b)
21. (b)	22. (d)		

Solutions

1. We know that

$$F = \frac{q_1 q_2}{(4\pi\varepsilon_0)r^2} \text{ and } E = \frac{F}{q}$$

$$\therefore \quad \varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

Hence $$\frac{1}{2}\varepsilon_0 E^2 = \frac{q_1 q_2}{8\pi F r^2} \times \frac{F^2}{r^2} = \frac{q_1 q_2}{8\pi q^2} \times \frac{F}{r^2}$$

$\therefore$ Dimensions of $\frac{1}{2}\varepsilon_0 E^2$ = dimensions of

$$\frac{F}{r^2} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

2. The capacitance of a parallel plate capacitor is given by $C = \varepsilon_0 A/d$. Hence the dimensions of $\varepsilon_0 L$ are the same as those of capacitance.

$\therefore$ Dimension of $\varepsilon_0 L \frac{\Delta V}{\Delta t}$

$$= \frac{\text{dimension of } C \times \text{dimensions of } V}{\text{time}}$$

$$= \frac{\text{dimension of } Q}{\text{time}} \qquad (\because Q = CV)$$

= dimension of current

3. The velocity of an electromagnetic wave is $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\therefore \quad \frac{1}{\mu_0 \varepsilon_0} = v^2 = [LT^{-1}]^2 = [L^2T^{-2}]$$

4. Momentum = mv = [ML T^{-1}]

Planck's constant $h = \frac{E}{V} = \frac{[ML^2 T^{-2}]}{T^{-1}} = [ML^2 T^{-1}]$

5. The exponent $\alpha Z/k\theta$ is dimensionless. Hence

$$[\alpha] = \left[\frac{k\theta}{Z}\right] = \left[\frac{JK^{-1} \times K}{m}\right] = J\,m^{-1} = [M\,L\,T^{-2}]$$

Dimensions of $\frac{\alpha^2}{\beta}$ are the same as those of P. Hence

$$[\beta] = \left[\frac{\alpha^2}{P}\right] = \frac{[MLT^{-2}]^2}{[ML^{-1}T^{-2}]} = [M\,L^3\,T^{-2}]$$

6. The energy stored in an inductor is given by

$$U = \frac{1}{2}LI^2$$

or $$L = \frac{2U}{I^2}$$

$\therefore$ Dimensions of inductance

$$(L) = \frac{\text{dimension of energy}}{\text{dimensions of (current)}^2}$$

$$= \frac{ML^2\,T^{-2}}{I^2} = \frac{ML^2\,T^{-2}}{(QT^{-1})^2} \qquad \left(\because I = \frac{Q}{T}\right)$$

$$= ML^2Q^{-2}$$

All other choices have dimensions different from ML^2Q^{-2}.

7. The proportionate error in the measurement of g is

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Hence Δg will be minimum if ΔL and ΔT are minimum. Thus the correct choice is (d).

8. $$Y = \frac{FL}{Al} = \frac{4MgL}{\pi d^2 l} \qquad (1)$$

Where

M = 1.0 kg (exact), g = 9.8 ms^{-2} (exact)

L = 2 m (exact), l = 0.8 mm = 0.8 $\times 10^{-3}$ m

Δl = ±0.05 mm, d = 0.4 mm = 0.4 $\times 10^{-3}$m

Δd = ±0.01 mm

Substituting the values of M, g, L, d and l in Eq. (1) we get

$$Y = 2.0 \times 10^{11}\ Nm^{-2}$$

From Eq. (1) the proportionate uncertainty in Y is given by

$$\frac{\Delta}{Y} = \frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

Since the values of M, g and L are exact, $\Delta M = 0$, $\Delta g = 0$ and $\Delta L = 0$. Hence

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta l}{l} = \frac{2\times 0.01\,\text{mm}}{0.4\,\text{mm}} + \frac{0.05\,\text{mm}}{0.8\,\text{mm}}$$
$$= 0.05 + 0.0625$$
$$= 0.1125$$
$$\therefore\ \Delta Y = 0.1125 \times Y = 0.1125 \times 2.0 \times 10^{11}$$
$$= 0.225 \times 10^{11}\ \text{Nm}^{-2}$$

Since the value of Y is correct only up to the first decimal place, the value of ΔY must be rounded off to the first decimal place. Thus $\Delta Y = 0.2 \times 10^{11}$ Nm^{-2}. Therefore, the result of the experiment is

$Y + \Delta Y = (2.0 \pm 0.2) \times 10^{11}$ NM^{-2}

9. Vernier constant = value of 1 M.S.D – value of 1 V.S.D.

   Now n V.S.D = $(n-1)$ M.S.D = $(n-1)\,x$ cm

   $$\therefore\ 1\ \text{V.S.D} = \left(\frac{n-1}{n}\right) x\ \text{cm}$$

   $$\therefore\ \text{V.C.} = x\ \text{cm} - \left(\frac{n-1}{n}\right) x\ \text{cm} = \frac{x}{n}\ \text{cm}$$

10. $T = 2\pi\sqrt{\dfrac{l}{g}} \Rightarrow g = \dfrac{4\pi^2 l}{T^2}$ Therefore,

    $$\frac{\Delta g}{g} = \frac{\Delta t}{l} + \frac{2\Delta T}{T}$$

    For student I, $E_{\text{I}} = \dfrac{\Delta g}{g}100 = \left(\dfrac{0.1}{64.0} + \dfrac{2\times 0.1}{128.0}\right) \times 100$

    $$= \frac{5}{16}\%$$

    For student II, $E_{\text{II}} = \left(\dfrac{0.1}{64} + \dfrac{2\times 0.1}{64}\right)\times 100$

    $$= \frac{15}{32}\%$$

    For student III, $E_{\text{III}} = \left(\dfrac{0.1}{20.0} + \dfrac{2\times 0.1}{36}\right)\times 100$

    $$= \frac{19}{18}\%$$

    Thus, the precentage error is minimum for student I.

11. Pitch of screw = $\dfrac{1\text{ mm}}{2} = 0.5$ mm

    Least count = $\dfrac{0.5\text{ mm}}{50} = 0.01$ mm

    Measured diameter = 3 mm + 35 × 0.01 mm
    = (3 + 0.35) mm = 3.35 mm

    Corrected diameter = measured diameter – zero error
    = 3.35 –(–0.03) = 3.38 mm

12. Momentum $p = m \times v = 3.513 \times 5.00 = 17.565$ kg ms^{-1}. The total number of significant figures in the result of multiplication or division is equal to the number of signficiant figures in the least accurate quantity. The velocity v has the least number of significant figures (equal to three). Hence, the result must be rounded off up to three significant figures as $p = 17.6$ kg ms^{-1}.

13. $F = q\,v\,B \sin\theta$. Therefore, dimensions of B are

    $$[B] = \frac{\text{dimension of } F}{\text{dimension of } qv} = \frac{\text{M L T}^{-2}}{\text{CL T}^{-1}} = [\text{MT}^{-1}\,\text{C}^{-1})$$

14. Value of 1 main scale division = 0.5º

    30 vernier scale divisions = 29 main scale divisions = 29 × 0.5º

    $\therefore$ Value of 1 vernier scale division = $\dfrac{29}{30}\times 0.5^{\circ}$

    Least count = value of 1 m.s.d – value of 1 v.s.d.

    $$= 0.5^{\circ} - \frac{29}{30}\times 0.5^{\circ} = \frac{0.5^{\circ}}{30} = \frac{0.5}{30}\times 60\,\text{min} = 1\ \text{min}$$

15. 23.023 has 5 significant figures (s.f.), 0.0003 = 3 × 10^{-4} has 1 s.f. and 2.1 × 10^{-3} has 2 s.f.

16. Least count = 1 M.S.D. – 1 V.S.D.

    $$= 1\ \text{M.S.D.} - \frac{16}{20}\ \text{M.S.D.}$$
    $$= \frac{4}{20}\ \text{M.S.D.}$$
    $$= \frac{4}{20} \times 1\ \text{mm} = 0.2\ \text{mm},$$

    which is choice (d).

17. Least count (L.C.) = $\dfrac{1\text{mm}}{100} = 0.01$mm = 0.001 cm

    Diameter of wire = main scale reading + (circular scale reading) × L.C.

    = 0 + 52 × 0.001

    = 0.052 cm

18. Least count of screw gauge

    $$= \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$
    $$= \frac{0.5\,\text{mm}}{50} = 0.01\ \text{mm}$$

    $\therefore$ diameter of ball = 2.5 mm + 20 × 0.01 mm = 2.7 mm

    Density $\rho = \dfrac{M}{\frac{4\pi}{3}r^3}$

    Maximum relative error in ρ is

    $$\therefore\ \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta r}{r}$$
    $$= 2\% + 3 \times \frac{0.01}{2.7}\times 100$$
    $$= 2\% + 1.1\% = 3.1\%$$

19. Least count = $\frac{0.5 \text{ mm}}{100}$ = 0.005 mm

$$\frac{\Delta Y}{Y} = \frac{\Delta \ell}{\ell} + \frac{2\Delta d}{d}$$

$$\frac{\Delta \ell}{\ell} = \frac{0.005\,\text{mm}}{0.25\,\text{mm}} = 0.02$$

$$\frac{2\Delta d}{d} = \frac{2 \times 0.005\,\text{mm}}{0.5\,\text{mm}} = 0.02$$

So, the correct choice is (a).

20. Least count = value of 1 MSD – value of VSD

$$= 0.05 \text{ cm} - \frac{2.45}{50} \text{ cm}$$

$$= 0.05 - 0.049 = 0.001 \text{ cm}$$

Diameter = 5.10 cm + 24 × 0.001 cm

= 5.10 cm + 0.024 cm

= 5.124 cm

21. Momentum = mv = $[\text{M L T}^{-1}]$

Planck's constant $h = \frac{E}{\nu}$

$$= \frac{[\text{M L}^2 \text{T}^{-2}]}{[\text{T}^{-1}]} = [\text{M L}^2 \text{T}^{-1}]$$

22. Speed of sound $v = \sqrt{\frac{\gamma RT}{M}}$. For a closed pipe, λ = $4L$ (fundamental mode). Also $v = \nu\lambda$. Thus

$$v = \nu \times 4L$$

$$\Rightarrow \quad \sqrt{\frac{\gamma RT}{M}} = \nu \times 4L$$

$$\Rightarrow \quad L = \frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}} \qquad (1)$$

For Neon: $M = 20 \times 10^{-3}$kg. Neon in monatomic gas for which $\gamma = 1.67$. Substituting the given values in (1), we have

[since $\sqrt{167\,RT}$ = 640 $\text{J}^{1/2}\,\text{mol}^{-1/2}$ and $\sqrt{\frac{10}{20}} = \sqrt{\frac{1}{2}} = \frac{7}{10}$]

$$L = \frac{1}{4 \times 244}\sqrt{\frac{1.67\,RT}{2 \times 10^{-2}}}$$

$$= \frac{1}{4 \times 244}\sqrt{\frac{167\,RT}{2}}$$

$$= \frac{1}{4 \times 244} \times 640 \times \frac{7}{10} = 0.459 \text{ m}$$

Similarly, for nitrogen $\gamma = 1.4$, L = 0.363 m, for oxygen $\gamma = 1.4$, $\lambda = 0.340$ m and for argon ($\gamma = 1.67$), L = 0.348 m. It is given that $L = (0.350 \pm 0.005)$m. Since the error in L is ± 0.005, the third digit (after the decimal point) in the value of L must be either zero or 5. Hence L = 0.348 m must be rounded off as L = 0.350 m. So the correct choice is (d).